MECHANICAL DRAWING.

MECHANICAL DRAWING.

PREPARED FOR THE USE OF THE STUDENTS

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BY

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MECHANICAL DRAWING.

CHAPTER I.

INSTRUMENTS AND THEIR USES.

1. To do good work, good instruments are essential. An accomplished draftsman *may* do fair work with poor instruments, but the beginner will find it sufficiently hard to do creditable work without being handicapped by poor instruments. It is also essential that the instruments should be kept in good order. They should be handled carefully, and wiped, before being put away, with wash-leather or chamois skin. This is especially needful if the hands perspire perceptibly.

2. Pencilling. Drawings should always be first made in pencil, and inked afterwards if desired. The idea of pencilling is to locate the lines exactly, and to make them of the required length. Accuracy in a drawing can only be obtained by accuracy in the pencil construction. There is a great tendency among beginners to overlook this important fact, and to become careless in pencilling, thinking they will be able to correct their inaccuracies when inking. This is a great mistake, and one to

be especially avoided. This accuracy can be obtained in pencil only by making very *fine*, *light* lines, and to this end hard pencils, 6 H, should be used, and they should be *kept sharp*. For drawing straight lines the pencil should be sharpened to a flat, thin edge, like a wedge. The compass pencils should be sharpened to a point. A softer pencil, 4 H, sharpened to a point, should be used in making letters, figures, etc.

It should be borne in mind that a 6 H pencil sharpened to a chisel point will make a depression in the paper, which can never be erased, if much pressure is put on the pencil; hence, press very lightly when using a hard pencil, so as to avoid this difficulty. If a drawing is not to be inked, but made for rough use in the shop, or where accuracy of construction (in the drawing) is not essential, or to be traced, a soft pencil would preferably be used, the lines being made somewhat thicker, or heavier.

3. COMPASSES. In using the compasses the lower part of the legs should be kept nearly vertical, so that the needle point will make only a small hole in revolving, and both nibs of the pen may press equally on the paper. In pencilling it is not so essential that the pencil point be kept vertical, but it is well to learn to use them in one way, whether pencilling or inking.

Hold the compasses loosely between the thumb and forefinger only, and do not press the needle point into the paper. If it is sharp, as it should be, the weight of the compass will be sufficient to keep it in place. While revolving, lean the compass very slightly in the direction of revolution, and put a little pressure on the pencil or pen point.

In removing the pencil or pen point to change them, be very careful to pull them out *straight;* do not bend them from side to side, in order to get them out more easily, as it would enlarge the socket and consequently spoil the instrument for accurate work.

In drawing a circle of larger radius than could be drawn with the compass in its usual form a *lengthening bar* is used. In this case steady the needle point with one hand and describe the circle with the other.

The large compasses are too heavy and clumsy to make small circles nicely, hence the bow compasses should be used in making all circles smaller than three-quarters of an inch radius, or thereabouts, depending on the stiffness of the spring. Be very careful to adjust the needle point to the same length as the pencil or pen point. In changing the radius of the bow compasses or spacers, press the points together, thus removing the pressure from the nuts, before turning the nuts in either direction. The screw thread will last much longer if this is done.

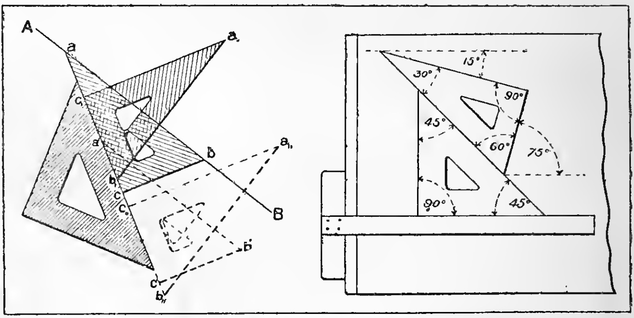
4. Dividers or Spacers. These are used to lay off distances from scales, or from other parts of a drawing to a line, or to divide a line into equal parts. In laying off the same distance several times on a line, keep one of the points of the dividers on the line all the time, and turn the instrument in an opposite direction each time, so that the moving point will pass alternately to the right and left of the line. Do not make holes in the paper in doing this, as it is impossible to ink nicely over them; a very slight puncture is sufficient.

5. T-Square. The T-square should be used with the head against the left-hand edge of the drawing board (unless the person is left-handed), and horizontal lines only should be drawn with it. Lines perpendicular to these should not be drawn by using the head of the T-square against an adjoining edge of the board, as there is no pains taken to make these edges at right angles to each other, but they should be drawn by using the triangle in connection with the T-square.

Lines should be drawn with the upper edge only of the T-square.

In case you wish to use the T-square as a guide for the knife in cutting paper to size, do not use the upper edge as a guide, but turn the T-square over and use the bottom edge. For, unless you are very careful, the knife will nick the edge, which would render it unfit to draw lines with.

6. Triangles. *To draw lines which shall be parallel to another by means of the triangles.* Let AB be the given line.



Place either edge of either triangle so as to coincide exactly with the given line. Place the other triangle (or any straight edge) against one of the other edges of the first triangle. Then, holding the second triangle, or straight edge, securely in this position with the left hand, move the first one, still keeping the two edges in contact. Any line drawn along the edge which originally coincided with the line AB will be parallel to it.

To draw lines which shall be perpendicular to another by means of the triangles. Let AB be the given line. Place the longest side of the triangle so as to coincide exactly with the given line.

Place the other triangle (or any straight edge) against one of the other edges of the first triangle. Then, holding the second triangle securely in this position with the left hand, revolve the first one so that its third edge is against the second triangle or straight edge. Any line drawn along the edge which originally coincided with the given line AB will be perpendicular to that line.

The right-hand portion of the figure shows how the two triangles may be used, in connection with the T-square, to draw lines making angles of 15° and 75° with a given line (in this case the line which coincides with the edge of the T-square). By turning the triangles over, these angles may be drawn in the opposite direction.

Lines making angles of 30°, 45°, and 60° are drawn directly by means of one triangle and T-square or straight edge.

7. Irregular Curves. To trace an irregular curve through a series of points, use that part of the edge of the curve which coincides with the greatest possible number of points (never less than three), and draw the curve through these points, then shift the curve so as to coincide with other points in the same way, letting the instrument run back on a part of the curve already drawn, so that a continuous smooth curved line may be formed.

It requires a considerable practice to draw irregular curves by means of an instrument, the tendency being to make a series of loops, on account of some of the points being covered up. There is no better way to put in a curve in pencil than by doing it free-hand, provided the hand and eye have been properly trained. Of course the curve cannot be inked in free-hand; the irregular curve must be used, but, being no longer confined to points, it is not difficult. V. P.

8. Scales. As it is frequently impossible to make a drawing on paper the real size of the object, it is customary to reduce the actual measurements by means of an instrument called a scale,— that is, the drawing may be made $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, etc., size, according as the relative size of the object and drawing may require.

If it is desired to make a drawing $\frac{1}{4}$ size, then 3 inches on the drawing will represent one foot on the object. It is frequently necessary to represent inches and fractions of an inch, hence divide the 3 inches into 12 equal parts, and each of these parts will represent one inch on the object. If each of the 12 parts are subdivided into 2, 4, or 8 parts, each part would represent respectively $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{1}{8}$ of an inch on the object. This may be designated, scale, 3 inches equal one foot, or $\frac{1}{4}$ size.

On the scale, one inch equal one foot, the unit, one inch, is divided into 12 parts to represent inches as before. Thus, to make a scale of any unit to one foot, it is simply necessary to divide that unit into 12 parts to represent inches, subdividing these parts, as far as possible, to represent fractions of an inch.

If the smallest division on a scale represents $\frac{1}{8}$ of an inch on the object, the scale is said to read to $\frac{1}{8}$ of an inch.

The student will find on his triangular scale ten different scales, viz., $\frac{3}{32}$, $\frac{1}{8}$, $\frac{3}{16}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{2}$, and 3 inches to the foot, reading to $3''$, $1''$, $1''$, $1''$, $1''$, $\frac{1}{2}''$, $\frac{1}{2}''$, $\frac{1}{4}''$, $\frac{1}{4}''$, and $\frac{1}{8}''$, respectively, (the double prime over a number or fraction means inches, the single prime indicates feet).

The scale should never be used as a ruler to draw lines with.

9. Needle Point. Each student should procure a *fine* needle, break off the eye end, and force the broken end into a small, round piece of soft pine wood. This is to be used in pricking off measurements from the scale, marking the exact

intersection of two lines, etc. Here, as in the case of the needle point in the compasses, it should not be forced into the paper: the finest puncture possible is sufficient.

10. Drawing Paper. This paper comes in sheets of standard sizes, as follows: —

Cap, ...	13 x 17 inches.	Elephant, . .	23 x 28 inches.
Demy, . . .	15 x 20 "	Columbia, . .	23 x 34 "
Medium, . . .	17 x 22 "	Atlas, . . .	26 x 34 "
Royal, . . .	19 x 24 "	Double Elephant,	27 x 40 "
Super-Royal, .	19 x 27 "	Antiquarian, .	31 x 53 "
Imperial, . .	22 x 30 "	Emperor, . .	48 x 68 "

Whatman's paper is considered the best. This paper is either hot or cold pressed, the hot pressed being smooth and the cold pressed rough. The rough paper is better for tinting work, the smooth takes ink lines better than the rough, but erasures show much more distinctly on it, hence the cold pressed is better for general work. The names of the sizes of the paper given above have no reference to quality. There is very little difference in the two sides of the paper, but that one which shows the maker's name in water lines, when held up to the light, is considered the right side.

11. Thumb Tacks. These are used for fastening the paper to the drawing board when it is not necessary to stretch it.

12. The geometrical problems in the next chapter are not given with the view of teaching geometry, but to give the student practice in the *accurate* use of his instruments.

In order that the degree of accuracy of the execution of the problems may be readily seen, these problems will not be inked.

13. The plates on which these problems are to be drawn should be laid out 10″ by 14″ (and cut this size when finished)

with a border line one inch from each edge. That portion of the plate within the border line is to be divided into 6 equal squares, in each of which one problem is to be drawn, beginning with No. 1 in the upper left-hand corner square, No. 2 in the upper middle, No. 3 in the upper right, etc.

The number of the plate is to be printed in the upper right-hand corner of the plate, about one-eighth of an inch above the border line, and the student's name in the lower right-hand corner, about one-eighth of an inch below the border line.

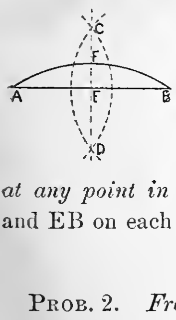
PROB. 1, 11, etc., as the case may be, is to be printed in the upper right-hand corner of each square. The initial letters of the name should be made $\frac{3}{16}$ of an inch high, and the small letters $\frac{1}{8}$ of an inch high.

The letters are to be made like the samples furnished in the drawing room, and they should be made as nicely as possible.

CHAPTER II.

GEOMETRICAL PROBLEMS.

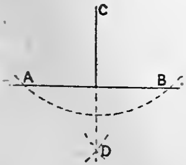
PROB. 1. *To bisect a straight line* AB, *or arc of a circle* AFB.



With A and B as centres and any radius greater than one half AB draw arcs intersecting in C and D. Join CD. CD is perpendicular to AB, and E and F are the middle points required.

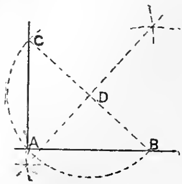
NOTE. *To draw a perpendicular to a line at any point in it, as* E *in* AB. Lay off equal distances EA and EB on each side of E, and proceed as above.

PROB. 2. *From a point* C *outside a straight line* AB *to draw a perpendicular to the line.*



With C as a centre and any convenient radius cut AB in the points A and B. With A and B as centres and any radius draw arcs intersecting in D. Join C and D, and CD is the perpendicular required.

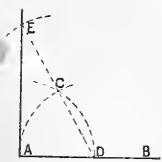
PROB. 3. *To draw a perpendicular to a line* AB *from a point* C *nearly or quite over its end.*



Draw a line from C to meet AB in any point B. Bisect BC in D by Prob. 1. With D as a centre and radius DC draw the arc CAB meeting AB in A. Draw through A and C.

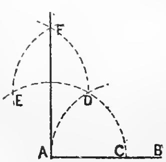
NOTE. If a perpendicular be required at A, take any point D as a centre and radius DA and draw an arc CAB. Through B and D draw a line to meet the arc in C. Draw through C and A.

PROB. 4. *To draw a perpendicular to a line* AB *from a point* A *at or near its end.*



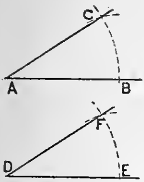
With A as a centre and any radius draw the arc CD. With centre D and same radius cut CD in C. With C as a centre and same radius draw an arc over A, and draw a line through D and C, producing it to meet this arc in E. Draw through A and E.

PROB. 5. *A second method.*



With centre A and any radius draw an arc CDE; with centre C and same radius cut this arc in D; with centre D and same radius draw arc EF; with centre E and same radius draw arc intersecting EF in F. Join AF.

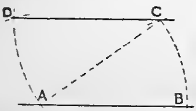
PROB. 6. *To construct an angle equal to a given angle* CAB.



Draw any line as DE, and take any point in it as D. With A as a centre and any radius cut the sides of the given angle in B and C. With centre D and same radius draw arc EF. With BC as a radius and E as a centre cut arc EF in F. Join DF.

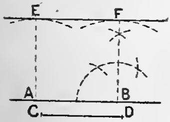
NOTE. For accuracy of construction in drawing this problem the longer the radius AB is the better.

PROB. 7. *Through a given point* C *to draw a line parallel to a given line* AB.



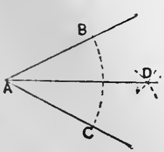
From C as a centre and any radius draw the arc AD; and from A as a centre, with the same radius, draw the arc BC. With BC as a radius, and A as a centre, draw an arc cutting the arc AD in D. Draw through D and C.

PROB. 8. *To draw a line parallel to a given line* AB *at a given distance* CD *from it.*



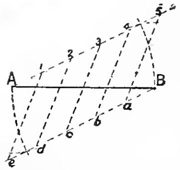
From any two points A and B on the line as centres and with CD as a radius draw arcs E and F. At A and B erect perpendiculars to meet the arcs in E and F. Draw a line through E and F.

PROB. 9. *To bisect a given angle* BAC.



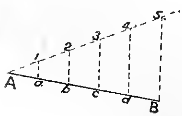
With A as a centre and with any radius draw the arc BC, cutting the sides of the angle in B and C. With centres B and C and any radius draw arcs intersecting in D. Draw AD.

PROB. 10. *To divide a given line* AB *into any number of equal parts.* (In this case 6.)



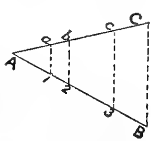
From A draw an indefinite line A, 1, 2 ·· 5 at any angle with AB. At B draw Ba ··· e, making the angle ABe equal to the angle BA5. With any distance as a unit, lay off on the lines from A and B as many equal spaces as the number of parts required less one. Join 1e, 2d, 3c, etc.

PROB. 11. *Another method. To divide a line* AB *into* (say) *five parts.*



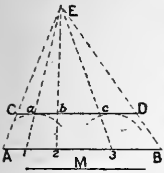
Draw A 1 ···· 5 at any angle to AB, and lay off on it five equal spaces, using any convenient unit. Join 5B, and through the points 1, 2, 3, 4 draw lines parallel to B5, meeting AB in points a, b, c, d.

PROB. 12. *To divide a line* AC *into the same proportional parts as a given divided line* AB.



Draw from a point A the lines AC and AB, making any angle. Join B and C. Through the points 1, 2, and 3 on AB draw lines parallel to BC, meeting AC in points a, b, and c, the required points of division.

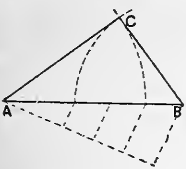
PROB. 13. *Second method.*



Let M be the line to be divided into parts proportional to the parts A1, 12, 23, and 3B of the line AB. Draw M parallel to AB at CD by Prob. 8. Draw lines through AC and BD to meet in E. Through E draw lines to 1, 2, and 3, cutting CD in *a*, *b*, and c, the required points of division.

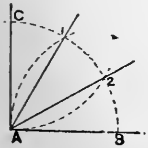
NOTE. If the parts on AB are equal, the parts on CD will be equal.

PROB. 14. *To draw on a given line* AB *as an hypothenuse a right triangle with its sides having the proportion of* 3, 4, *and* 5.



Divide AB by Prob. 11 into 5 equal parts. With centre B and a radius equal to three of the parts draw an arc, and with centre A and a radius equal to four of the parts cut this arc in C. Join AC and BC.

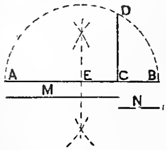
PROB. 15. *To trisect a right angle* CAB.



With centre A and any radius draw the arc of the quadrant, cutting the sides in C and B. With centres C and B and the same radius cut the arc in points 1 and 2. Join A1 and A2.

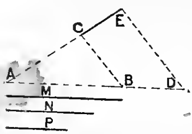
NOTE. The angle 1AB is an angle of 60°, the construction of which is apparent.

PROB. 16. *To find a mean proportional between two lines* M *and* N.



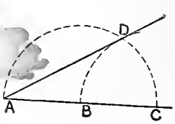
Upon an indefinite line AB lay off AC equal to M, and CB equal to N. Bisect AB in E by Prob. 1, and with centre E and radius EB draw a semicircle. At C draw CD perpendicular to AB (Prob. 5). CD is the mean proportional required.

PROB. 17. *To find a fourth proportional to three lines* M, N, *and* P.



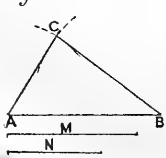
Draw AB equal to M, and at any convenient angle draw AC equal to N. Upon AB produced make BD equal to P. Join BC, and draw DE parallel to BC, to meet AC produced in E. CE is the fourth proportional required.

PROB. 18. *At a point* A *in a line* AC *to make an angle of* 30°.



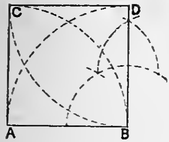
With any point B as a centre and radius AB draw a semicircle ADC. With centre C and same radius cut this arc in D. Draw AD. DAC = 30°.

PROB. 19. *Having given the sides* AB, M, *and* N *of a triangle to construct the figure.*



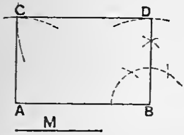
With centre A and radius M draw an arc. With B as a centre and radius N draw an arc to cut the first arc at C. Join AC and BC.

PROB. 20. *On a given side* AB *to construct a square.*



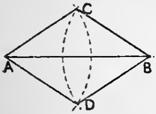
Draw BD at right angles to AB and equal to AB (Prob. 5). With A and D as centres and radius AB draw arcs intersecting in C. Join AC and CD.

PROB. 21. *To construct a rectangle of given sides* AB *and* M.



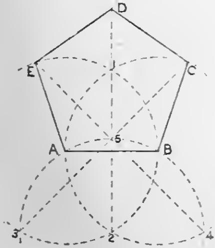
At B draw BD perpendicular to AB by Prob. 5, and equal to M. With A as a centre and radius equal to M draw an arc, and from D as a centre and a radius equal to AB cut this arc in C. Join AC and CD.

PROB. 22. *On a given diagonal* AB *to construct a rhombus of given side* AC.



With centres A and B and radius AC draw arcs intersecting in C and D. Join AC, AD, BC, and BD.

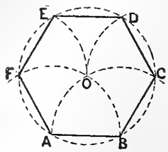
PROB. 23. *On a given base* AB *to construct a pentagon.*



With centres A and B and radius AB draw circles intersecting in 1 and 2. Join 1 and 2. With centre 2 and same radius draw the circle 3A5B4, giving points 3 and 4. Produce 35 to C and 45 to E. With centres C and E and radius AB draw arcs intersecting in D. Draw BCDEA.

NOTE. This is an approximate method.

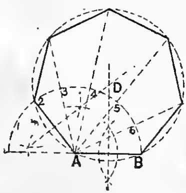
PROB. 24. *To construct a regular hexagon of given side* AB.



With A and B as centres and radius AB draw arcs intersecting in O. With centre O and radius AB draw a circle, and lay off BC, CD, etc., each equal to AB. Join the points B, C, D, E, F, and A.

NOTE. The radius of any circle goes around the circumference as a chord six times exactly.

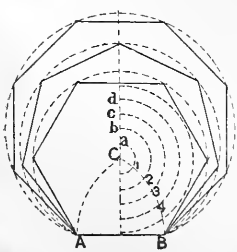
PROB. 25. *On a given base* AB *to construct a regular polygon of any number of sides* (in this case 7).



With centre A and radius AB draw a semicircle and divide it at points 1, 2, 3, 4, etc., into as many equal parts as there are sides in the required polygon. Draw a line from the second point of division 2 to A.

2A is one side of the required polygon. Bisect AB and A2 by perpendiculars, by Prob. 1, meeting in D. With D as a centre and radius DA draw the circle BA2, etc. Apply AB as a chord to the circle as many times as there are sides in the polygon.

PROB. 26. *On a given line* AB *to construct a polygon of any number of sides. An approximate method.*

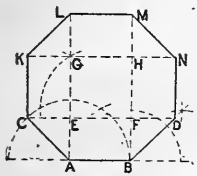


Bisect AB, and produce the bisecting line indefinitely. With centre A and radius AB draw the arc BC, cutting the bisecting line in C. Divide the arc BC into *six* equal parts, in points 1, 2, 3, etc.

To construct a pentagon. With centre C and radius C1 draw an arc, cutting the bisecting line in a point below C,

which is the centre of the circle circumscribing the pentagon. If the polygon is to have *more* sides than *six*, set *up* from C on the line C*ab* as many parts of the arc CB as added to *six* make the number of sides of the required polygon ; thus, for a seven-sided polygon set up *one* division as C*a*; for an eight-sided set up *two* divisions as C*ab*, and so on. *a*, *b*, *c*, *d*, etc., are the centres for the circumscribing circles of the polygons, each side of which is equal to AB.

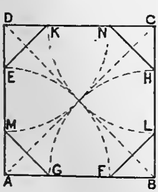
PROB. 27. *On a given base* AB *to construct a regular octagon.*



At A and B erect perpendiculars by Prob. 5 to AB, and bisect the exterior right angles, making the bisectors AC and BD each equal to AB. Draw CD, cutting the perpendiculars in E and F. Lay off EF from E to G and from F to H. Draw an indefinite line through GH. Make GK, GL, HN and HM each equal to CE or FD. Connect C, K, L, M, N, and D.

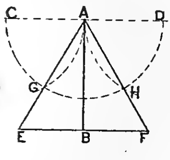
NOTE. This may be done by Prob. 25. Check the construction by seeing if AN, CM, BD, and KL are parallel. CA, KB, etc. should be parallel ; so also AD, CN, and KM.

PROB. 28. *To construct an octagon within a square* ABCD.



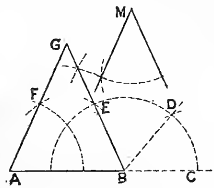
Draw the diagonals AC and BD. With A, B, C, and D as centres and the half of the diagonal of the square as a radius draw the arcs EF, GH, KL, and MN. Join the points MG, FL, HN, and KE.

PROB. 29. *The altitude* AB *of an equilateral triangle being given, to construct the triangle.*



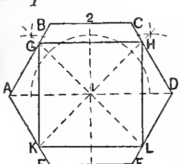
Draw CD and EF, both perpendicular to AB. With A as a centre and any radius describe the semicircle CGHD. With C and D as centres and the same radius draw arcs cutting the semicircle in G and H. Draw AGE and AHF.

PROB. 30. *Given the base* AB *of an isosceles triangle, and the angle at the vertex* M, *to draw the triangle.*



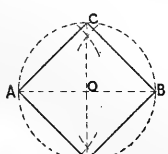
With centre B and any radius draw a semicircle cutting the base produced at C. Make the angle DBC equal to M by Prob. 6. Bisect ABD by Prob. 9. Make the angle FAB equal to EBA, and produce AF and BE to meet in G.

PROB. 31. *Within a regular hexagon* ABCDEF *to inscribe a square.*



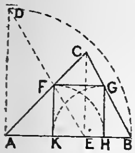
Draw a diagonal AD. Bisect AD by a perpendicular 213 (Prob. 1). Bisect by Prob. 9 the right angles 21A and 21B, and produce the bisectors to meet the sides of the hexagon in points G, H, L, and K. Join G, H, L, and K.

PROB. 32. *On a given diagonal* AB *to construct a square.*



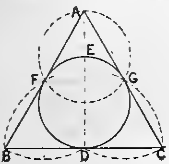
Bisect AB in O (Prob. 1); and with centre O and radius OA draw a circle to cut the bisecting line in C and D. Draw AC, CB, BD, and DA.

PROB. 33. *Within a given triangle* ABC *to inscribe a square.*



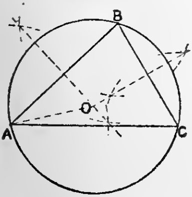
Draw AD perpendicular to and equal to AB. From C draw CE perpendicular to AB (Prob. 2). Draw DE cutting AC in F. From F draw FK perpendicular to AB. Make KH equal to KF, and from H with radius HK cut BC in G. Join FG and GH.

PROB. 34. *About a given circle* FEGD *to circumscribe an equilateral triangle.*



Draw the diameter DE. With centre E and radius of the given circle draw the circle AFG. Prolong DE to A. With centres D, F, and G and radius DG draw arcs intersecting at B and C. Join AB and AC.

PROB. 35. *To circumscribe a circle about a triangle* ABC.

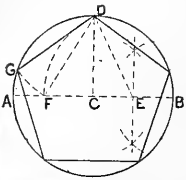


By Prob. 1 bisect two of the sides AB and BC by perpendiculars meeting in O. With centre O and radius OA draw the circle.

NOTE 1. If any three points are given not in the same straight line, a circle is passed through them by joining the points and proceeding as above.

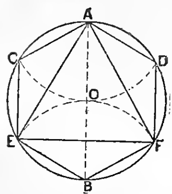
NOTE 2. If any circle is given, its centre is found by taking any three points in its circumference, and proceeding as above.

PROB. 36. *To inscribe a pentagon within a circle.*



Draw any diameter AB, and a radius CD perpendicular to it. Bisect BC in E. With centre E and radius ED draw the arc DF. With centre D and radius DF draw the arc FG. DG is the side of the required pentagon.

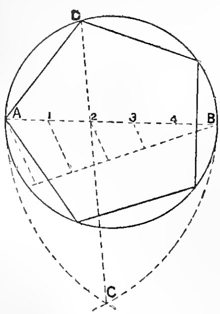
PROB. 37. *To inscribe a hexagon within a circle.*



Draw the diameter AB, and with centres A and B and the radius of the given circle draw arcs COD and EOF, cutting the circumference in C, D, E, and F. Join AD, DF, FB, etc.

NOTE. Joining points A, E, and F gives an inscribed equilateral triangle.

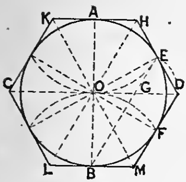
PROB. 38. *To construct a regular polygon of any number of sides, the circumscribing circle being given.*



Draw any diameter AB and divide it into as many equal parts as there are sides in the required polygon (in this case 5). With A and B as centres and radius AB draw arcs intersecting in C. Draw a line from C through the second point of division of AB to meet the circumference in D. AD is one side of the required polygon. Lay off AD as a chord as many times as there are sides in the required polygon (in this case 5).

NOTE. This is an approximate method.

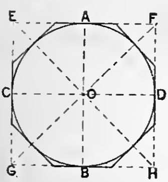
PROB. 39. *To circumscribe a hexagon about a circle.*



Draw the diameters AB and CD perpendicular to each other. Divide each quadrant into thirds, by Prob. 15, at points E, F, etc. Join B and E, cutting CD in G. With G as a centre and radius GE draw arc ED, cutting COD in D. With centre O and radius OD cut the diameters produced in points H, K, C, L, and M. Join points H, D, M, L, C, and K.

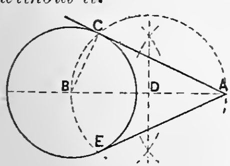
NOTE. Practically the 60° triangle placed on a T-square whose blade is parallel to COD will give HD, DM, etc., by making it tangent to the circle at E. F, etc. KH and LM are drawn by the T-square.

PROB. 40. *To circumscribe a square, also an octagon, about a circle.*



Draw the diameters AB and CD at right angles to each other. With centres A, B, C, and D and radius OA describe arcs intersecting in points E, F, G, and H. These points connected give a square about the circle. Inscribe an octagon in the square by Prob. 28.

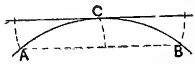
PROB. 41. *To draw a tangent to a circle* B *from a point* A *without it.*



Draw AB and bisect it in D. With centre D and radius DA draw a semicircle cutting the given circle in C and E. Join AC. By joining AE a second tangent is found, equal to AC.

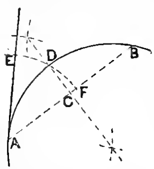
NOTE. To draw a tangent to a circle from a point C on the circumference. Join BC, and at C draw AC perpendicular to BC. *For the tangent is perpendicular to the radius at the point of tangency.*

PROB. 42. *To draw a tangent to the arc of a circle when the centre is not accessible.*



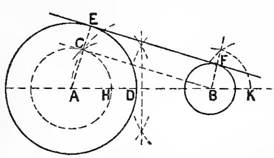
Let C be the point upon the given arc, AB, at which the tangent is to be drawn. Lay off equal distances upon the arc from C to A and B. Join A and B. Through C draw a line parallel to AB by Prob. 7.

PROB. 43. *To draw a tangent at a given point* A *on a circle when the preceding method is not applicable.*



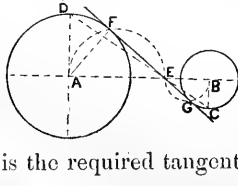
From A draw any chord AB. Bisect AB in C, and the arc ADB in D by Prob. 1. With A as a centre and a radius AD draw the arc EF. With D as a centre and radius DF draw an arc cutting EF at E. Join AE.

PROB. 44. *To draw a tangent to two given circles,* A *and* B.



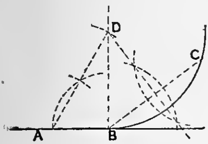
Through A and B draw a line. Make DH equal to the radius BF. Draw the circle A–HC, and from B draw the tangent BC by Prob. 41. Draw AC and produce it to E. Make the angle FBK equal to CAH. Join EF.

PROB. 45. *To draw a tangent to two given circles which shall pass between them.*



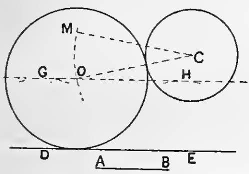
Join A and B, and draw AD and BC perpendicular to AB. Draw DC, cutting AB in E. Draw a tangent from E by Prob. 41 to the given circles. Join the tangent points F and G. FG is the required tangent.

PROB. 46. *To draw a circle tangent to a given line* AB *at a given point* B *in it, which shall also pass through a fixed point* C *without the line.*



Draw BD perpendicular to AB, at the point B. Join CB, and draw a perpendicular to it at its middle point, Prob. 1. The intersection of this perpendicular and BD gives D the centre required.

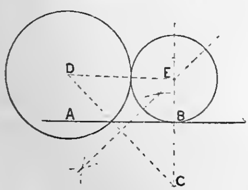
PROB. 47. *To draw a circle of a given radius* AB, *which shall be tangent to a given circle* C, *and also to a straight line* DE.



Draw GH parallel to DE, by Prob. 8, at the distance AB. With a radius CM, equal to the radius of circle C plus AB, draw an arc to meet GH in O. With the centre O and radius AB draw the required circle.

NOTE. If two circles are tangent, the straight line connecting the centres passes through the point of tangency. This point it is very important to locate precisely in all cases of tangency.

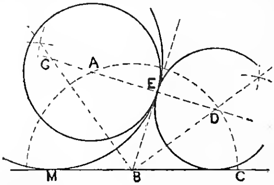
PROB. 48. *To draw a circle tangent to a given circle* D, *and also tangent to a given line* AB, *at a given point* B *on the line.*



Draw BC perpendicular to AB, and make BC equal to the radius of D. Join DC, and at its middle point draw a perpendicular to meet BC in E, the required centre.

NOTE. By laying off BC above the line AB, and proceeding as above, another circle is found below AB.

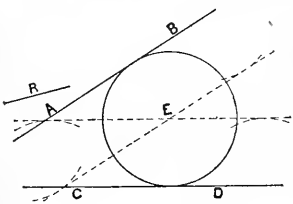
PROB. 49. *To draw a circle tangent to a given circle* A, *and a given line* BC, *at a given point* E *on the circle.*



Draw AE and produce it. At E draw a perpendicular to AE, meeting BC in B. Draw the bisector of the angle EBC, meeting AE produced in D. DE is the radius required.

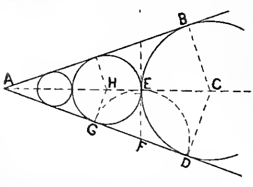
NOTE. By bisecting EBM another circle, whose centre is G, is found, *enclosing* the circle AE.

PROB. 50. *To draw a circle of given radius* R *tangent to two given lines*, AB *and* CD.



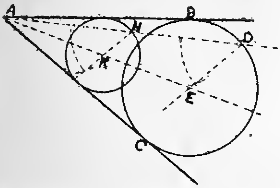
Draw lines parallel to AB, and CD, at the given distance R from them by Prob. 8, meeting in E, the required centre.

PROB. 51. *To draw any number of circles tangent to each other, and also to two given lines* AB *and* AD.



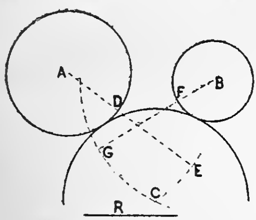
Bisect BAD by AC, Prob. 9. Let one of the circles be BED, drawn by taking C as a centre, and a radius equal to the perpendicular CD from C to AD. At E draw EF perpendicular to AC. With centre F and radius FE draw the arc DEG, cutting AD in G. At G make GH perpendicular to AC. H is a centre required. Repeat the process.

PROB. 52. *To draw a circle through a given point* D *and tangent to two given lines* AB *and* AC.



Draw AD. Bisect BAC by AE. From any point K, on AE as a centre, draw a circlé tangent to AB and AC, and cutting AD in H. Draw HK. At D draw DE, making the angle ADE equal to AHK. E is the required centre.

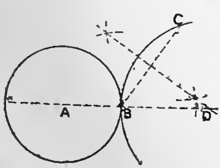
PROB. 53. *To draw a circle of a given radius* R *tangent to two given circles* A *and* B.



Through A and B draw indefinite lines, and make DE and FG each equal to R. With A and B as centres, and radii AE and BG, draw arcs cutting each other at C, the required centre.

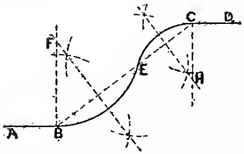
NOTE. These arcs will intersect in a second centre.

PROB. 54. *To draw a circle through a point* C, *and tangent to a given circle* A, *at a point B in its circumference.*.



Draw AB and produce it. Join BC and bisect it by a perpendicular meeting AB produced in D, the required centre.

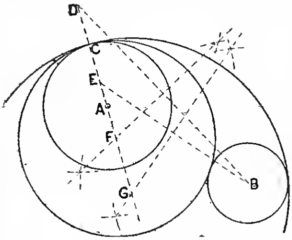
PROB. 55. *Given two parallel straight lines* AB *and* CD, *to draw arcs of circles tangent to them at* B *and* C, *and passing through* E, *which is anywhere on the line* BC.



At B and C erect perpendiculars. Bisect BE and CE by perpendiculars (Prob. 1), meeting the perpendiculars from B and C in F and H, the required centres. Draw the arcs BE and CE.

NOTE. This is called a reversed curve.

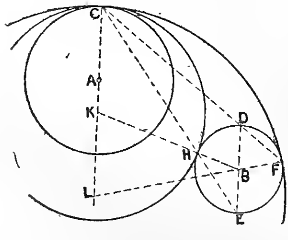
PROB. 56. *To draw a circle tangent to two given circles* A *and* B *at a given point* C *in one of them.*



Draw a line indefinitely through A and C. Make CD and CE each equal to the radius of B. Join BE and BD. Bisect BE and BD by perpendiculars meeting AC produced in F and G, the centres of the required circles.

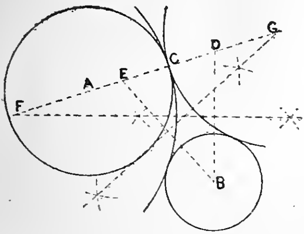
NOTE. The tangent points should be determined accurately.

PROB. 57. *Same as Prob.* 56. *A second method.*

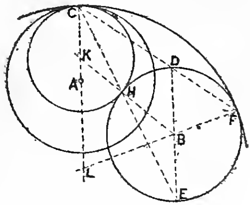


Draw a line through A and C indefinitely. Through B draw DE parallel to CA, cutting B in D and E. Draw CD to F and CE, cutting B in H. Through B and H draw BHK, cutting CA in K, one centre required. Through B and F draw FBL, cutting CA in L, another centre required.

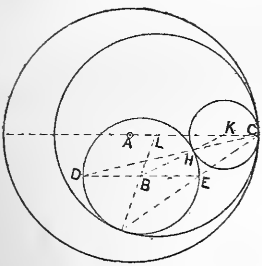
Prob. 58. *Same as Prob.* 56.

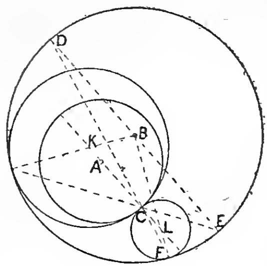


Prob. 59. *Same as Prob.* 56. *Method of Prob.* 57.

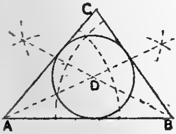


Probs. 60 and 61. *Same as Prob.* 56. *Method of Prob.* 57.



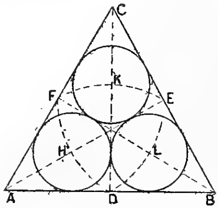


Prob. 62. *To inscribe a circle within a triangle* ABC.



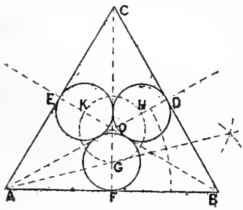
Bisect two of the angles of the triangle, say A and B, by Prob. 9. The bisectors meet in D, the centre of the required circle. A perpendicular from D to either side is the required radius.

PROB. 63. *Within an equilateral triangle* ABC *to inscribe three equal circles, each touching two others, and two sides of the triangle.*



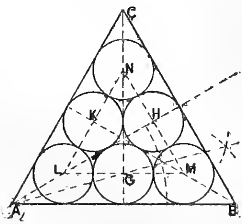
Draw the bisectors of the angles A, B, and C, cutting the sides in D, E, and F. With centres D, E, and F, and radius DF, draw arcs cutting the bisectors in H, L, and K, the required centres.

PROB. 64. *Within an equilateral triangle to draw three equal circles, each touching two others and one side of the triangle.*



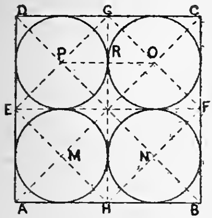
Bisect the angles A, B, and C. Bisect the angle DAB by AG. G is the centre of one of the required circles. With the centre O of the triangle as a centre, and radius OG, draw a circle cutting AD in H and BE in K. With centres G, H, and K, and radius GF, draw the circles G, H, and K.

PROB. 65. *Within an equilateral triangle* ABC *to draw six equal circles which shall be tangent to each other and the sides of the triangle.*



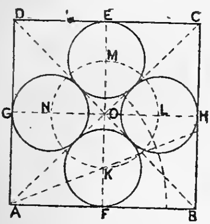
Inscribe the three circles K, H, and G by Prob. 64. Draw LGM parallel to AB, MHN parallel to BC, and LKN parallel to AC. The points L, M, and N are the centres of the other three circles.

PROB. 66. *Within a square* ABCD *to draw four equal circles each touching two others and two sides of the square.*



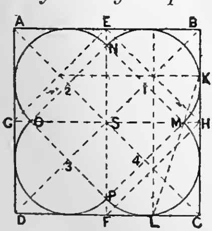
Draw the diagonals AC and BD, and the diameters EF and GH. Draw EH, HF, FG, and GE, giving the centres M, N, O, and P. The radius OR is found by joining OP.

PROB. 67. *Within a given square* ABCD *to draw four equal circles each touching two others and one side of the square.*



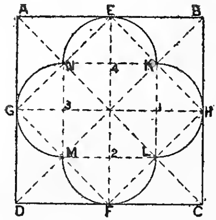
Draw the diagonals AC and BD, and the diameters EF and GH. Bisect the angle OAB by AK, cutting EF in K. With radius OK and centre O draw a circle cutting the diameters in the points L, M, N, and K, the required centres.

PROB. 68. *Within a square* ABCD *to draw four equal semicircles, each tangent to two sides of the square, and their diameters forming a square.*



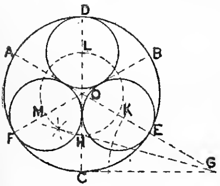
Draw the diagonals and diameters. Bisect FC and BH in L and K. Draw LK, cutting GH in M. Set off SM from S on the diameters to N, O, and P. Join the points M, N, O, P. The intersections of these lines with the diagonals give the required centrés 1, 2, 3, and 4.

PROB. 69. *Within a square* ABCD *to draw four equal semicircles, each touching one side of the square and their diameters forming a square.*



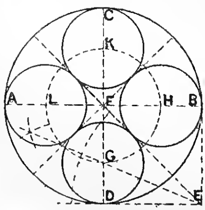
Draw the diagonals and diameters. Draw EH, HF, FG, and GE, giving the points K, L, M, and N. The lines joining these points cut the diameters in points 1, 2, 3, and 4, the required centres.

PROB. 70. *To draw within a given circle* ABC *three equal circles tangent to each other and the given circle.*



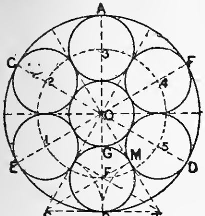
Divide the circle into six equal parts by diameters AE, DC, etc. (Prob. 24). Produce any diameter, as AE to G, making EG equal to the radius of the given circle. Join CG. Bisect the angle OGC by GH, intersecting OC in H. With centre O and radius OH draw the circle HKLM. K, L, and M are the centres of the circles.

PROB. 71. *To draw within a given circle* ACDB *four equal circles which shall be tangent to each other and the given circle.*



Draw the diameters AB and CD at right angles, and complete the square FBED. Draw the diagonal FE and bisect the angle FED by EG; FD and EG meet in G. With centre F and radius FG draw a circle cutting the diameters in G, H, K, and L, the required centres.

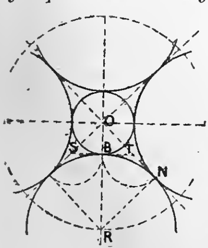
PROB. 72. *Within a given circle* AFD.... C *to draw six equal circles tangent to each other and the given circles.*



Draw the diameters AB, CD, and EF, dividing the circle into six equal parts (Prob. 24, Note). Divide any radius as OB into three equal parts (Prob. 11), at points F and G. With centre O and radius OF draw a circle giving the centres F, 1, 2, 3, 4, 5 required. A circle of the radius FB may be drawn from centre O tangent to the six circles.

NOTE. The above is a special method. *The general method, to draw any number of equal circles in a given circle, tangent to each other and the given circle,* is to divide the circle by diameters into twice as many equal parts as circles required. From B, the extremity of any one of these diameters, draw a tangent ST, Prob. 41, Note. Produce the diameters on each side of AB to meet the tangent in S and T. Bisect the angle T. The bisector meets OB in F, the centre of one of the required circles. Or F may be obtained by making TM equal to TB, and at M drawing a perpendicular to OT, meeting OB in F. With centre O and radius OF draw a circle cutting the diameters in points 1, 2, 3, 4, etc., the centres required.

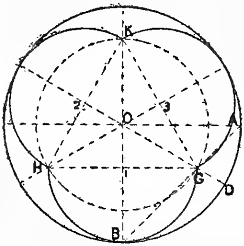
PROB. 73. *About a given circle to circumscribe any number of equal circles tangent to each other and the given circle.*



Divide the circumference of the given circle by diameters into twice as many parts as circles required. From the extremity B of any diameter draw a tangent SBT (Prob. 41, Note), and produce the diameters on each side of OB to meet SBT in S and T. Produce OT, making TN equal to TB. Make NR perpendicular to TN, meeting

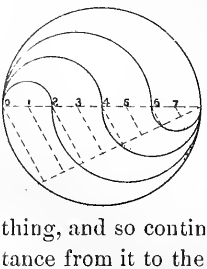
OB produced in R, the centre of one of the required circles. The other centres are at the intersection of a circumference drawn with radius OR and centre O, and every other diameter produced.

PROB. 74. *Within a given circle* AC . . . E *to draw any number of equal semicircles touching the given circle, and their diameters forming a regular polygon.*



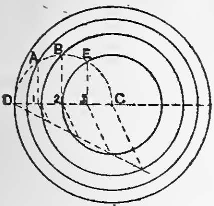
In the given circle let OA and OB be two radii at right angles. Divide the given circumference, commencing at B, into twice as many parts as semicircles required, and draw diameters to the points of division. Join BA. BA cuts the first diameter to one side of OB at G. G is one end of two adjacent diameters required. Lay off OG from O on *every other* diameter in points H, K, etc. Join HK, KG, etc. 1, 2, 3, etc., are the centres of the required semicircles.

PROB. 75. *To divide a circle into any number of parts which shall be equal in area.*



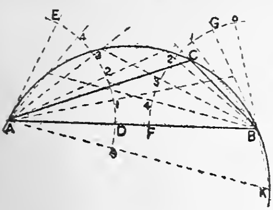
Let the number of parts be four. Divide a diameter into twice as many equal parts as areas required, in this case eight, by points 1, 2, 3, 4, etc. With 1 and 7 as centres and radius 01 describe a semicircle on opposite sides of the diameter; with centres 2 and 6 and radius 02 do the same thing, and so continue taking each point as a centre and the distance from it to the end of the diameter as a radius.

PROB. 76. *To divide a circle into concentric rings having equal areas.*



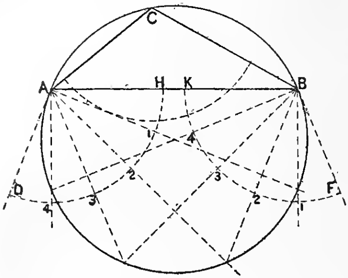
Divide the radius CD into as many equal parts as areas required (Prob. 11) in points 1, 2, 3, etc. On CD as a diameter draw a semicircle, and at the points 1, 2, 3, etc. draw lines perpendicular to CD, meeting the semicircle in points A, B, etc. With centre C and radii CA, CB, etc. draw concentric circles.

PROB. 77. *A chord* AB *and a point* C *being given, to find other points in the arc of the circle passing through* A, B, *and* C *without using the centre.*



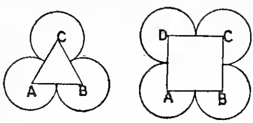
Draw AB, AC, and BC. Suppose four more points are required. With any radius and centres A and B draw the arcs DE and FG. Make CAE equal CBA, and GBC equal CAD. Divide the arcs DE and FG into the same number of equal parts, one more than the number of points required.. Number the points 1, 2, 3, 4, from D toward E, and from G toward F. Draw lines from A and B through these points, and those passing through like-numbered points meet in points on the arc ACB. To construct a point on the curve below AB lay off G0 equal to G1 on the arc FG, and D9 equal to D1. Lines through 0 and B, and A and 9 meet in K, a point on the curve.

PROB. 78. *On a chord* AB *to construct the supplementary arc to* ACB, *without using the centre,* C *being a point on the arc.*



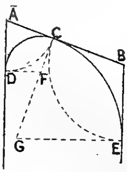
Join AC, AB, and BC. Make BAD and ABF each equal to ACB. With A and B as centres, and any radius, draw arcs HD and KF. Divide the arcs DH and KF into the same number of equal parts (say five) by the points 1, 2, 3, etc. Number the points as shown. Draw lines from A through 1, 2, 3, etc. to meet lines through the same numbers drawn from B. The lines through like numbers meet in points on the required arc.

PROB. 79. *To construct any number of tangential arcs of circles, having a given diameter.*



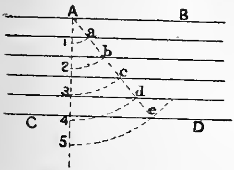
Suppose three (four) arcs are required. Upon the given diameter as a base draw an equilateral triangle ABC (square ABCD). With each vertex as a centre, and a radius of half a side, draw arcs of circles tangent to each other, as shown.

PROB. 80. *At a point* C *on a line* AB *to draw two arcs of circles tangent to* AB, *and to two parallels* AD *and* BE, *forming an arch.*



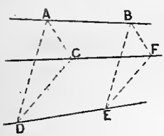
Make AD equal to AC, and BE equal to BC. At C make CG perpendicular to AB, and at D and E draw the perpendiculars DF and GE, meeting CG in F and G, the required centres.

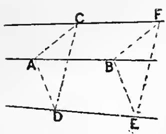
PROB. 81. *To draw any number of equidistant parallel straight lines between two given parallels,* AB *and* CD.



Draw any line A5 at any angle to AB, and lay off on it any convenient distance A1 as a unit as many times as lines required, *plus* one. Thus, if four parallels are required, lay off A1 *five* times. With A as a centre and A5 as radius draw an arc cutting CD in point *e*. Join A*e*. With centre A and radii A1, A2, A3, etc., cut A*e* in *a*, *b*, *c*, etc. Through *a*, *b*, *c*, etc. draw parallels to AB and CD.

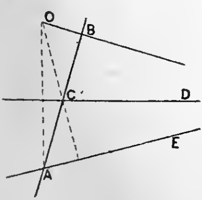
PROB. 82. *To draw through a point* C *a line to meet the inaccessible intersection of two lines,* AB *and* DE.





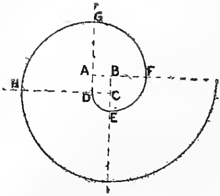
From C draw any lines CA and CD to the given lines. Join AD, AC, and CD. Make BE parallel to AD, BF parallel to AC, and EF parallel to CD. The intersection of EF and BF gives a point F in the required line. Draw through C and F.

PROB. 83. *To draw a perpendicular to a line* AB, *which shall pass through the inaccessible intersection of two lines,* AE *and* CD.



Produce AB to cut the lines in A and C. From A draw a perpendicular to CD, and from C a perpendicular to AE; these perpendiculars meet in O. Draw BO perpendicular to AB. OB passes through the intersection of CD and AE.

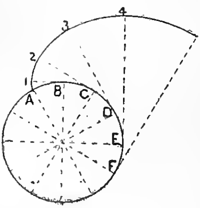
Prob. 84. *To draw an involute of a square* ABCD.



Produce the sides as shown. With centre C and radius CD draw the arc DE. With centre B and radius BE draw the arc EF. With centre A and radius AF draw the arc FG, etc.

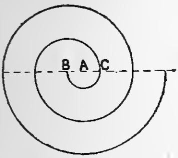
Note. Suppose a line to be wrapped around and in the direction of the perimeter of any plane figure. Let the line be *unwound*, keeping it always straight in the process of unwinding. Any point in the line describes an *involute.* The involute of polygons is composed of arcs of circles, as in Prob. 84.

Prob. 85 *To draw the involute of a circle.*



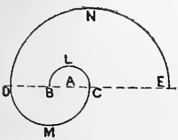
Divide the circumference into any number of equal parts, as at A, B, C, D, etc., and draw radii to these points. At A, B, C, D, etc. draw tangents. Let the curve start at A. On the tangent at B lay off a distance from B to 1 equal to one of the parts into which the circumference is divided. On the tangent at C lay off a distance equal to two parts to 2. On the tangent at D three parts to 3; from E four parts to 4, etc. The curve through these points, 1, 2, 3, 4, etc., is the involute of the circle.

PROB. 86. *To draw a spiral composed of semicircles, the radii being in arithmetical progression.*



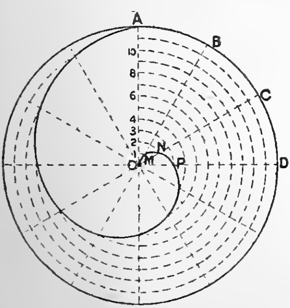
Draw an indefinite line, BAC. On the line take any two points, A and B, as centres. With A as a centre and radius AB draw a semicircle. With B as a centre and radius BC draw a semicircle, and so on, using A and B as centres, and taking the radii to the end of the diameter of the last-drawn semicircle.

PROB. 87. *To draw a spiral composed of semicircles, whose radii shall be in geometrical progression.*



Let the ratio be 2. Let AB be the radius of the first circle, A its centre. Draw the semicircle BLC. With B as a centre and radius BC draw the semicircle CMD. With C as a centre and radius CD draw the semicircle DNE. D is the next centre, the diameter DE of the last-drawn circle becoming the radius for the next circle. So proceed.

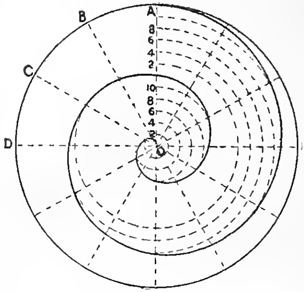
PROB. 88. *To draw a spiral of one turn in a given circle.*



Divide the circle into any number of equal parts, say twelve, by the lines OA, OB, OC, etc., and a radius OA into the same number of equal parts by the points 1, 2, 3, 4, etc. With O as a centre and radius O1 draw an arc cutting OB in M; with centre O and radius O2 an arc cutting OC in N, radius O3 an arc cutting OD in point P, etc. Through M, N, P, etc. draw the curve.

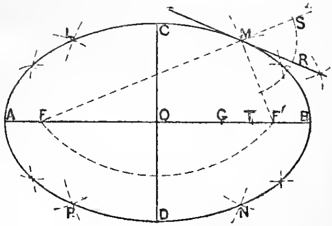
NOTE. This is "the spiral of Archimedes."

PROB. 89. *To draw in a given circle a spiral of any number of turns, say two.*



Draw radii dividing the circle into any number of equal parts. Divide any radius, as OA, into as many equal parts as turns in the spiral, and divide each part into as many equal portions in points 1, 2, 3, 4, etc. as the circle is divided into. With centre O and O1, O2, O3, etc. as radii, draw arcs to meet the radii OB, OC, OD, etc. in points of the required curve.

PROB. 90. *Given the axes* AB *and* CD *of an ellipse to draw the curve, and at any point on the curve to draw a tangent.*

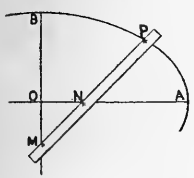


Place the axes AB and CD at right angles to and bisecting each other at O. With centre C and radius OA cut AB in F and F′, which are the foci. Between O and F′, or F, take any point G, dividing AB into two parts. With centres F and F′ and radius AG draw arcs on either side of AB. With the same centres and radius BG draw arcs intersecting those drawn with radius AG, at points L, M, N, and P, which are points on the curve. Take any other point, T on AB, and repeat the above operation; and so on until as many points as are necessary are found. Through the points draw the curve. FM plus F′M equal AB. Let M be the point at which the tangent is required. Produce FM and draw the bisector of the angle SMF′. MR is the tangent required.

NOTE. For drawing the ellipse and similar curves through a series of points the so-called French Curves are to be used.

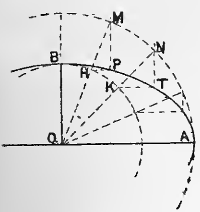
NOTE. The major axis is sometimes called the *transverse*, and the minor the *conjugate*, axis.

PROB. 91. *To draw an ellipse by means of a trammel, the axes being given.*



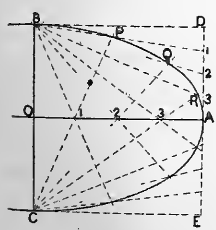
Let the semi-axes be OA and OB. Mark off on the straight edge of a slip of paper or card MP equal to OA, and NP equal to OB. Keep the trammel with the point N always on the major axis, and the point M on the minor axis, and P will be a point in the curve. Find as many points as necessary, and draw the curve.

PROB. 92. *To draw an ellipse, having given the axes.*



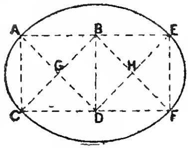
Let the semi-axes be OA and OB. With radii OA and OB and centre O draw circles. Draw *any* radii, OM, ON, etc. Make MP, NT, etc. perpendicular to OA, and HP, KT, etc. parallel to OA. P, T, etc. are points on the curve.

PROB. 93. *To draw an ellipse, having given the axes.*



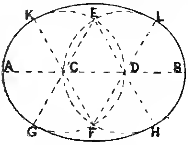
Place the axes at right angles at their centres, and on them construct a rectangle, one half being shown in BDEC. Divide OA and DA into the same number of equal parts by points 1, 2, 3, etc. Draw lines through C and 1, 2, 3, etc. to meet lines from B drawn to 1, 2, 3, etc. on AD. P, Q, R, etc. are points on the curve.

PROB. 94. *To draw a curve approximating to an ellipse.*



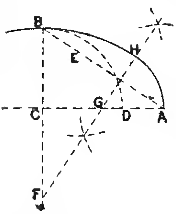
Draw the squares ABDC and BEFD, and their diagonals, intersecting in G and H. With centres G and H and radius GA draw the arcs AC and EF. With centres B and D and radius DA draw the arcs CF and AE.

PROB. 95. *To draw on a given line,* AB *as a major axis, a curve approximating an ellipse.*



Divide AB into three equal parts (Prob. 11) by points C and D. With centres C and D and radius CA draw two circles intersecting in E and F. Through C and D draw ECG, EDH, FCK, and FDL, meeting the circles in points G, H, K, and L. With centres E and F and radius EG draw the arcs GH and KL, completing the curve.

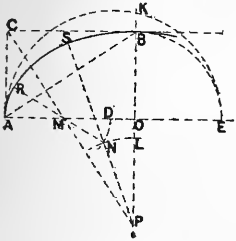
PROB. 96. *Given the major and minor axes of an ellipse, to draw the curve approximately.*



Let CA be the semi-major axis, and BC the semi-minor axis. Join A and B. Make CD equal to BC, and BE equal to AD. Bisect AE by a perpendicular, meeting BC produced in F. With centre F and radius FB draw the arc BH, and with centre G and radius GH draw the arc HA.

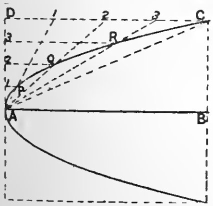
NOTE. One quarter of the whole curve is only shown, leaving to the student the construction of the full ellipse.

Prob. 97. *Having the axes given, to draw a curve of tangential arcs of circles approximating to the ellipse.*



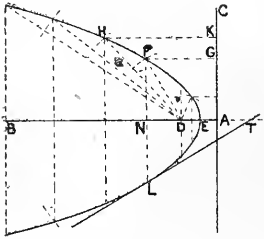
AO is the semi-major axis, and OB the semi-minor axis. Draw a rectangle with the axes as sides. AOBC is one-quarter of the rectangle. Draw AB. From C draw CMP perpendicular to AB, and meeting BO produced in P. Make OE equal to OB. On AE as a diameter draw a semicircle AKE. Produce OB to K. Make OL equal to BK. With centre P and radius PL draw the arc LN. Make AD equal to OK, and with centre M and radius MD draw the arc DN, meeting LN in N. Draw NMR and PNS. With centre M and radius MA draw AR; with centre N and radius NR draw RS, and with centre P and radius PS draw an arc from S through B. Repeat in each of the quadrants.

Prob. 98. *To draw a parabola when the abscissa* AB *and the ordinate* BC *are given.*



Draw the rectangle ABCD, and divide AD and DC into the same number of equal parts. Through the points of division on AD draw parallels to AB, and from A draw lines to the points on DC. The first line above AB meets the line from A to the first point of division from D in a point P on the curve. The second parallel to A meets the second from A to DC and so on. P, Q, R, and C are points in the curve. Repeat the same below AB.

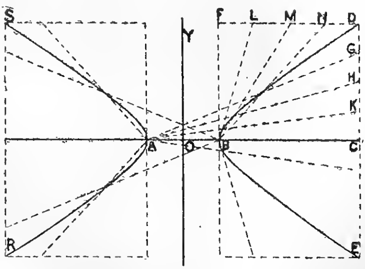
PROB. 99. *To draw a parabola when the directrix* AC *and the focus* D *are given, and to draw a tangent at any point* L *on the curve.*



This curve is such that its apex E is always half way between A and D, and the distance from D to any point upon the curve, as F, is always equal to the horizontal distance from F to the directrix. Thus DF equals FG, and DH equals HK, etc. Through D draw BDA perpendicular to AC. This is *the axis* of the curve. Draw parallels to AC through any points in AB, and with centre D and radii equal to the horizontal distances of these parallels from AC cut the corresponding verticals, which will give points on the curve.

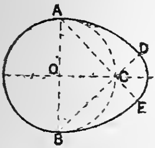
To draw the tangent at L. Draw the ordinate LN, meeting AB in N. Produce BA to the right. Make ET equal to EN. Draw LT, the tangent required.

PROB. 100. *To draw an hyperbola when the diameter* AB, *the abscissa* BC, *and the double ordinate* DE *are given.*



Complete the rectangle BCDF, and divide CD and DF each into the same number of equal parts. Draw BL, BM, and BN, intersecting lines AK, AH, and AG respectively in points on the curve. Repeat below and in the other half of the curve as indicated.

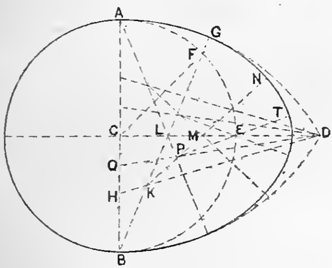
PROB. 101. *To draw an oval on the diameter of a given circle.*



Let AB be the diameter. Draw the circle ACB. Make OC perpendicular to AB. Draw the lines BCD and ACE indefinitely. With centres A and B and AB as a radius draw the arcs BE and AD. With centre C and radius CD draw the arc DE.

NOTE. The centres A and B may be taken anywhere on the line AOB produced.

PROB. 102. *Upon a given line* AB *to draw an oval.*



Bisect AB at C, and draw the perpendicular CD. With B as a centre and radius AB describe an arc AD. Bisect the quadrant AE in F. Through F draw BFG. AG is the first part of the curve. Bisect CB in H, and draw HD. K is the second centre. Bisect EL in M, and draw KMN. GN drawn from K is the second part of the curve. Bisect CH in O, and draw DO. P is the third centre. From P through E draw PET. NT is the third part of the curve. From E with a radius ET carry the curve to the line DC, and repeat the operation for the other half of the curve. Draw a semicircle on the diameter AB for the other part of the oval.

The cycloid is the path described by any point in the circumference of a circle which rolls along a straight line.

An epicycloid is the path described by any point in the circumference of a circle which rolls along the outside of another circle.

A hypocycloid is the path described by any point in the circumference of a circle which rolls along the inside of another circle.

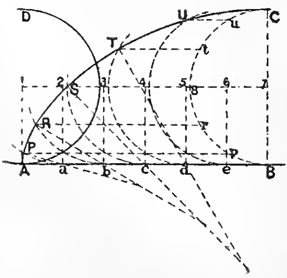
The rolling circle is called the generatrix, or generating circle, and the line (straight or curved) on which it rolls is called the directrix.

Every point in the tire of a wheel which rolls along the ground in a straight line describes a cycloid.

Hence it is easily seen that in one revolution or turning around of the circle, or wheel, the circumference will roll out into a straight line. To lay off the circumference on the straight line, either calculate its length or divide the circle into equal arcs, and lay off on the straight line as many divisions as there are in the circle; each division on the straight line being equal to the length of one division on the arc.

The arc is more than the chord, so a distance greater than the chord must be taken, and this is obtained by judgment or by approximation. By dividing the arc into a number of smaller arcs, so that the chord practically coincides with the arc, the small chord being laid off as many times as there are small arcs, a length of straight line is obtained very nearly equal to the arc given.

PROB. 103. *To construct a cycloid.*



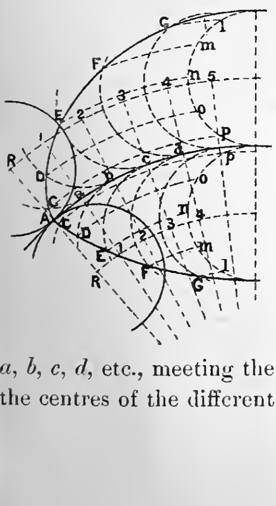
Let AB be the directrix, and AD the generating circle. Divide the rolling circle into any number of equal parts, say 12, and lay off these lengths of arcs along AB, giving points a, b, c, etc. Through 1, the centre of the generating circle, draw a line parallel to AB. This is the line of centres. On

this lay off 12, 23, 34, etc., equal to A*a*, *ab*, etc., and with centres 2, 3, etc. draw the generating circle in all its positions tangent to AB at points *a*, *b*, *c*, etc. Draw through the points of division on the rolling circle parallels to AB, to meet the different positions of the rolling circle in points P, R, S, T, and U. These parallels are drawn in the figure from points *p*, *r*, *s*, *t*, and *u*. Repeat the process for the other half of the curve.

Another method is to take the chords of the arcs B*p*, B*r*, B*s*, etc., and with centres *a*, *b*, *c*, etc. cut the respective circles in points P, R, S, etc. The chord *a*P equals B*p*; *b*R equals B*r*; *c*S equals B*s*, etc.

NOTE. When the number of divisions of the rolling circle is large the curve may be drawn by arcs of circles by taking *a* as a centre, and radius *a*A, and drawing from A to P. Produce P*a* and R*b* to meet, giving the centre for arc PR; R*b* and S*c* meet at the centre of arc R*s*, etc.

PROB. 104. *To construct an exterior epicycloid.*



Let R–A be the rolling circle on the *outer* circumference of the directing circle. Divide R–A into any number of equal parts (say 12), and lay off these parts on A*ab*, etc., giving points *a*, *b*, *c*, *d*, etc.

With the centre of the directing circle as a centre, draw an arc from R giving the line of centres R123 etc. Draw from the centre of the directing circle radial lines through *a*, *b*, *c*, *d*, etc., meeting the line of centres in points 1, 2, 3, etc., the centres of the different positions of the rolling circle. With

centres 1, 2, 3, etc. and radius RA draw the several positions of the rolling circle. With the centre of the directing circle as a centre, draw arcs through the points of division of the circle R–A to meet the several positions of the rolling circle in points C, D, E, F, etc., which are points on the curve. Draw through C, D, E, F, etc.

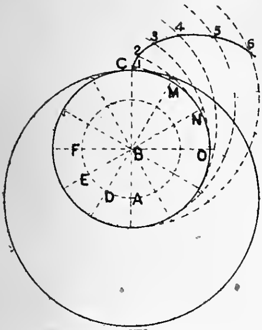
The points of the curve may be obtained by drawing from *a* as a centre and radius equal to the chord of one division of R–A an arc to meet the second position of the rolling circle in C; from *b* with radius equal to the chord of two divisions an arc to meet the third position in D; from *c* with radius equal to the chord of three divisions to meet the fourth position in E, etc. Or from *a* lay off on the rolling circle tangent at *a one* part of R–A (in this case $\frac{1}{12}$); on the one tangent at *b two* parts of R–A; on the one tangent at *c three* parts, etc.

Prob. 105. *To construct a hypocycloid.*

See the curve on the interior of the directing circle in figure with Prob. 104. A*a*, *ab*, *bc*, etc. are equal parts of the circumference of the rolling circle. R123 etc. is the line of centres. The process and directions are the same as for the epicycloid.

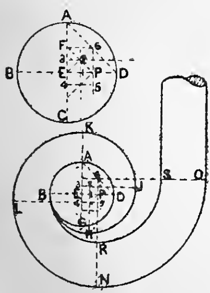
Note. When the diameter of the rolling circle is equal to the radius of the directing circle the hypocycloid becomes a straight line.

Prob. 106. *To construct an interior epicycloid.*



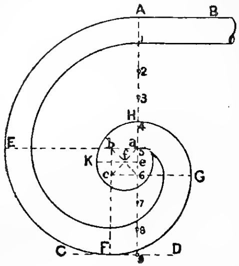
Let the circle A–C roll on B–CMN. On the circumference of B–C lay off equal arcs CM, MN, NO, etc. Draw from M, N, O, etc. radial lines through B, and make MD, NE, OF each equal to the radius CA. With centres D, E, F, etc. draw the circles tangent at M, N, O, etc. Lay off M1 equal to CM, giving point 1; from N lay off *two* divisions each equal to MC, giving point 2; from O lay off *three* spaces, giving point 3, etc. 1, 2, 3, etc. are points of the curve.

Prob. 107. *To draw a scroll for a stair-railing.*



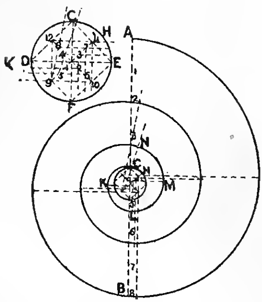
The circle ABCD is the eye of the scroll. Draw the diameters AC and BD at right angles. Draw the chord AD, and bisect it in 6. Draw a line 65 parallel to AC. Bisect AE in F. Bisect EF in 3. Make E4 equal to E3. Draw 32 and 45 parallel to BD, and 21 parallel to AC. From 6 draw an indefinite line parallel to BD and produce 65, 21, etc. From point 1 with radius 1B draw the arc BH. From 2 and radius 2H draw HJ, and so on. The arc BR of the inner curve is drawn from P with radius PB; the arc RS is drawn from 6 with radius 6R.

PROB. 108. *Another method.*



Suppose the scroll to come between the outside of the rail AB and the line CD. Draw A9 perpendicular to AB and CD, and divide it into nine equal parts. On the sixth division as a side draw the square *abc*6. With centre *a* and radius *a*A draw the quadrant AE. With centre 6 and radius 6E draw arc EF. With centre *c* and radius *c*F draw FG; with centre 6 and radius 6G draw arc GH; with centre *e* and radius *e*H draw arc HK; and with centre *f* and radius *f*K draw the arc KL. For the inner curve start with *a* as a centre and radius *a*1 and proceed as for the outer.

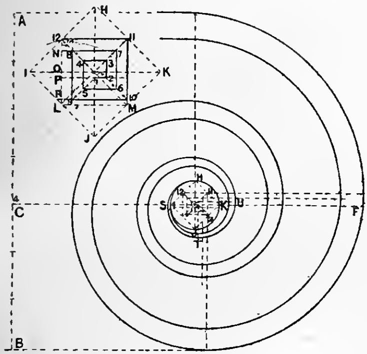
PROB. 109. *To construct a spiral, its greatest diameter* AB *being given.*



Divide AB into eight equal parts by points 1, 2, 3, etc. On 45 as a diameter draw a circle CDEF. This circle is the eye of the spiral. Inscribe a square CDEF, as shown in the enlarged drawing of the eye. Draw the central diameters of the square, 12,10 and 11,9. Divide these diameters into six equal parts, and number as shown. With 1 as a centre and radius 1H draw an arc HK to meet a horizontal produced from 2 through 1. With 2 as a centre and radius 2K draw the arc KL to meet a vertical through 3 and 2. With centre 3 and radius 3L draw arc LM meeting a horizontal produced through 4 and 3. With 4 as a centre and radius 4M draw arc MN to meet a line drawn through 5 and 4 at N, and so proceed.

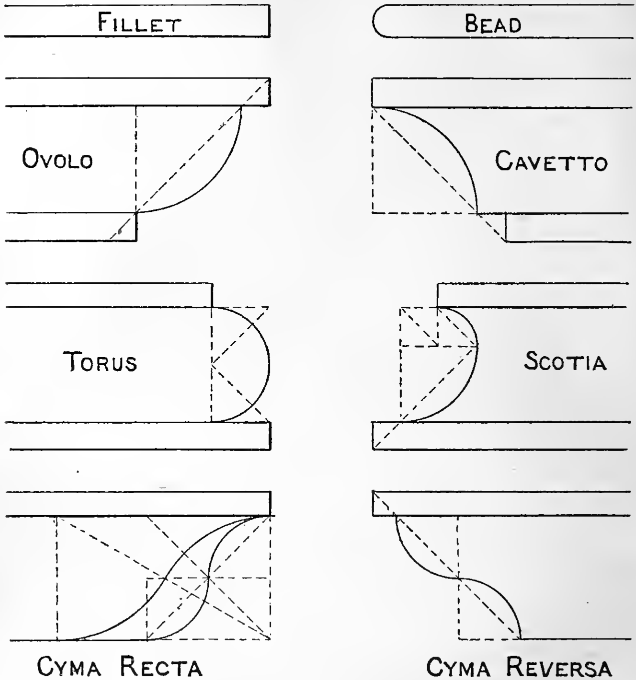
The curve may be commenced by taking 12 as a centre and radius 12 A and drawing from without toward the centre.

PROB. 110. *To describe an Ionic volute.*



Let AB be the vertical measure of the volute. Divide AB into seven equal parts, and from C, the lower extremity of the fourth division, draw CF perpendicular to AB, of indefinite length. From any point on CF as a centre, with a radius equal to one-half of one of the divisions of AB, draw the circle HIJK, forming the eye of the volute. Draw the diameter HJ perpendicular to CF. Draw the square HIJK, bisect its sides, and draw the square 12 L M 11. Draw the dividing lines of the square as shown in the smaller figure, and extend them. The divisions corresponding to 12 N are equal. The divisions OP and RL are each equal to one-half of 12 N. From 1 as a centre and radius 1H draw the arc HS; from 2 as a centre and radius 2S the arc ST; from centre 3 and radius 3T the arc TU; and so proceed in the order of the numerals.

In drawing the inner curves the dots on the diagonals in the small figure indicate the centres. The division of the square, of which 12 N is one side, shows how these centres are found.



The Roman moldings are given above, the method of construction being evident. All the arcs are arcs of circles, and the angles are 45°, except in the flatter form of the cyma recta, where the line of centres is at 30°.

CHAPTER III.

INKING.

14. It is supposed that the student has now become familiar with the use of the instruments necessary for the construction of a drawing in pencil, and has acquired a certain degree of proficiency in handling them which is necessary for accurate work. The next step is to learn to ink a drawing after it has been pencilled.

Starting with a good pen, in good condition. and a smooth, well-ground black ink. it only remains for the student to learn to make a clean, sharp, even line. This may seem at first like an easy thing to do; nevertheless, the ability to make a good ink line every time comes to most students very slowly, and after a great deal of practice. Therefore, before beginning to work on the plates, which are to be finished carefully and handed in for inspection, it will save a considerable time and paper to make lines against the triangle without regard to their length, direction, or location, until the student is thoroughly familiar with his pen and can make a fair line. Several hours, if necessary, in this preliminary practice will be invaluable.

15. India Ink. A special ink, called India ink, is always used in making drawings. It comes either in the stick form, and has to be ground as used, or in the liquid form. In the

latter form it is held in solution by an acid which corrodes the pen and eats into the fibre of the paper, so that if it is desired to erase a line it is much more difficult than if made with the ground ink. This ink is also very liable to rub off like soot. The only advantage it has is the saving of time in preparation. This kind of ink cannot be recommended for anything except coarse, rough work. It should not be used for tinting.

To prepare the stick ink for use place a small amount of water in the ink slab or saucer (the slab should be perfectly clean), then grasp the ink firmly and, with a rotary motion, grind until the liquid is *black* and a little sluggish in its motion. After it has reached the point when it is black enough the grinding should cease, as a continuation only makes the liquid thicker, thus causing it to flow less freely from the pen. The liquid will *look* black in the slab after a very little grinding, but the necessary consistency will not be reached for some time. In order to determine when the proper point is reached make a heavy line with the drawing pen on a piece of paper and wait for it to dry; do not go over this line a second time. If the ink has not been ground sufficiently, it will look pale after it is dry, in which case more grinding is necessary. If at any time the ink becomes too thick, it can be diluted by putting in more water and mixing thoroughly. The stick should always be wiped dry after using, to prevent its crumbling. The ground ink should be kept covered as much as possible to prevent evaporation, which would soon cause it to become too thick for use. It is not advisable to prepare a large quantity of ink at once, as the greater the amount the longer it takes, and freshly ground ink is preferable. If carefully covered, however, it may be kept two or three days. In case the ink becomes dry in the saucer it should all be washed out, as it is almost impossible to redissolve it entirely so that there will not be little scales which get into the pen and cause the ink to flow irregularly.

16. Drawing Pen. This instrument, commonly called a right-line pen, is one of the most important of the drawing instruments, and it is very essential that it be of good quality. The screw is used to adjust the distance between the nibs, in order to make the line of the desired weight. The ink may be placed between the nibs by means of a brush or strip of paper, but it is more convenient to dip the pen into the ink, being careful to wipe the outside of the nibs before using.

While inking the pen should be held so that both nibs rest on the paper evenly, and it should be inclined a little to the right, or in the direction of the line, with its flatter side against the triangle or straight edge, the end of the middle finger resting on the head of the screw. A *slight* downward pressure is necessary (the greater the rougher the paper), but do not press against the ruler, as the lines would be uneven in thickness. The ruler is simply a guide for the pen. The lines should always be drawn from left to right (relative to the person and not to the drawing). If it is desirable to go over a line a second time for any reason, it should be drawn in the same direction; never go backward over it.

In inking a curved line by means of the right-line pen and irregular curve, it is necessary to constantly change the direction of the pen so that the nibs shall always be tangent to the curve. This requires considerable practice to do nicely.

In case the ink does not flow freely from the pen, moisten the end of the finger and touch it to the end of the pen, and try it on a piece of waste paper. If this fails the pen should be wiped out clean and fresh ink put in. In making fine lines the nibs of the pen are near to each other, consequently the ink dries between them quite rapidly, hence it will be found advisable to clean out the pen thoroughly quite frequently to insure perfect lines. This is one of the secrets of being able to make

good fine lines; they should also be made more rapidly than heavy lines; the heavier the line the slower the pen should be moved. Do not keep the point of the pen too near the straight edge, as the ink is liable to flow against it, thus causing a blot. Especial attention should be given to the care of the pens; they should always be carefully wiped after using, and should not be put away with any ink dried on them, nor allowed to get rusty on the inside of the nibs. Any old piece of cotton cloth will answer to wipe the pens and stick of ink on.

17. How to Sharpen the Pen. To make good lines the pen must be kept in first-class condition,—that is, not only clean, but sharp; and every draftsman should be able to sharpen his own pen.

The curve at the point of the nibs of the pen should always be a semi-ellipse, with its long diameter coinciding with the axis of the pen; it should not be a semicircle, nor should it be pointed. The student is advised to look carefully at the points of his new pen, so as to get a correct idea of the proper curve before it becomes changed by wear. When this curve becomes changed by wear, or if, from any other cause, one nib is longer than the other, the nibs should be screwed together, then, holding the pen in a plane perpendicular to the oil-stone, draw it back and forth over the stone, changing the slope of the pen from downward and to the right to downward and to the left, or *vice versa*, for each forward or backward movement of the pen, so as to grind the points to the proper curve, making them also of exactly the same length.

This process, of course, makes the points even duller than before, but it is a necessary step. Next separate the points a little by means of the screw, and then place either blade upon the stone, keeping the pen at an angle of about 15° with the

face of the stone, move it backward and forward, at the same time giving it an oscillating motion, until the points are sharp. This is quite a delicate operation, and great care should be exercised at first. The pressure upon the stone should not be very great, and it is well to examine the point very often so as to be sure and stop when each nib has been brought to a perfect edge, otherwise one nib is liable to be longer than the other and the pen will not work well, even if each nib is sharp of itself.

Although the points want to be brought to a perfect edge, they should not be sharp like a knife, as in that case they would cut the paper. It will probably be necessary to try the pen with ink to be sure that it is in good condition. Sometimes a slight burr is formed on the inside of the blades; this is removed by separating the points still farther, so as to insert the knife-edge of the oil-stone between them and draw it carefully through. One motion should be sufficient to remove the burr.

The pen should never be sharpened by grinding the inside of the blades other than just indicated.

18. Inking a Drawing. In inking a drawing it is preferable to ink all the circles and arcs first, as it is easier to make the straight lines meet the arcs than the reverse. Of a number of concentric circles the smallest should be inked first. Here, as in the case of the pencil compasses, the pen point should be kept nearly vertical, the top of the compass being inclined a little toward the direction of revolution, and there should be a slight downward pressure on the pen point, but none on the needle point.

Where a large number of lines meet at a point care should be taken to avoid a blot at their intersection. These lines should be drawn from rather than toward the point, and each line should be thoroughly dry before another is drawn.

In case of two lines meeting at a point neither line should stop before reaching the point, nor go beyond it. Either of these defects gives a very ragged appearance to the drawing.

19. Stretching Paper. For ordinary small line drawings it is usually sufficient to fasten the paper to the board by means of thumb tacks, but for large drawings, or those which are to be tinted at all, it is necessary to stretch the paper by wetting it and fasten it to the board with mucilage. To do this lay the paper on the board, fold over about one-half an inch along each edge of the sheet; do not cut the corners; next wet the upper surface of the paper, except that portion folded over; do not *rub* the surface of the paper, simply press the sponge against it on all parts; apply the mucilage to one of the edges and fasten that edge down, beginning at the middle and rubbing toward either end; do the same with the opposite edge next, giving a slight pull to the paper as it is fastened down; repeat this process for the two remaining edges.

It is very important that the edges of the paper where the mucilage is to be applied should be kept dry, so that the mucilage will be ready to act as soon as it can dry, and to facilitate this the less mucilage you can use and accomplish the result the better. If a large quantity of mucilage is used it will moisten the edges of the paper so much that it will be likely not to stick, as the body of the paper will dry as soon as the edge, and therefore pull them up. The drying of the mucilage can be hastened by rubbing the edge briskly with a piece of thick paper under the fingers until it becomes hot. The board should never be placed near the fire or radiator to hasten the drying, as it would dry the paper before the mucilage set, causing the edges to be pulled up. The board should be left to dry in a horizontal position, and all the superfluous water should

be removed with a sponge, so as to avoid water marks in the paper, which always show in tinted drawings. In some cases, when the mucilage sets slowly, it may be necessary to moisten the centre of the paper sometime after stretching, to prevent its pulling up the edges by drying too rapidly.

20. Correcting and Cleaning Drawings. Pencil lines are removed by means of a piece of rubber. When a mistake is made in inking, or it is desired to change a completed drawing, it becomes necessary to erase an ink line. This can be done by means of a rubber ink eraser, the same as in the case of pencil lines, except that much more rubbing is necessary. Ink lines can also be removed, and more quickly, by means of a knife; in this case care should be taken not to use the point of the knife, as V-shaped holes are made which will always show. The flat portion of the knife should be used. After erasing an ink line, the surface which has been made rough by scratching should be rubbed down with some hard, perfectly clean, rounded instrument before inking other lines over it.

A drawing can be cleaned by means of India rubber, or stale bread crumbled on the drawing and rubbed over it. Although dirt *can* be removed from a drawing, it should be the aim of the draftsman to keep it as clean as possible. Therefore, the drawing should be kept covered when not being worked upon, and, if the drawing is a large one, all except that portion which is in use should be kept covered.

TINTING.

21. Tinting may be done in colors or India ink, as desired. The method of putting on the tint is the same in either case; consequently, we will take up only the India-ink tint here.

If a drawing is to be tinted, the paper must be stretched as

explained in the first part of this chapter. Especial care should be taken to keep the paper perfectly clean. That portion of the drawing which is to be tinted must not be touched with the India rubber, as the surface is thereby made rough and will not take a uniform tint. Hence, in laying out the work pencil lines must not be made on the surface to be tinted.

22. Preparation of the Tint. Clean the ink slab, water glass, and the brushes thoroughly, also be sure that there are no scales on the stick of ink which could possibly come off. Fill the slab about half full of water, and grind the ink as previously explained until it is black, but not thick. Fill the water glass about half full of clean water, and with a brush transfer enough of the ink in the slab to the glass to make a light tint. It is hard to get an ink which is *absolutely* free from specks, therefore, it is well to let the ink, after it is prepared in the slab, stand a short time to allow these specks to settle to the bottom; then, in transferring the ink to the glass, do not plunge the brush down to the bottom of the slab, thus taking up this sediment, but let the brush fill from the surface of the liquid.

The mixture in the water glass is the one to be used for tinting, and it is better not to make it as dark as you wish it upon the drawing when finished, as it is much easier to put on a light tint evenly than a dark one. The required depth of shade can be obtained by successive washes. Let each wash dry thoroughly before putting on another. A smoother effect can usually be obtained, especially on a large surface, by going over the surface to be tinted with clean water first, and then letting it dry.

23. Laying on the Tint. Having laid out the surfaces to be tinted, incline the board so as to slope like an ordinary

desk; then dip your brush into the tint you have mixed, and take up as much of the liquid as it will carry, begin in the upper left-hand corner of the surface and draw it along the upper boundary, holding the brush nearly vertical and leaving quite a puddle as you proceed. Lead this puddle gradually downward by going across the surface from left to right, about a quarter of an inch at a time, dipping the brush frequently in the tint so as to keep the puddle about the same size all the time; it should not be large enough to run down at any point. This puddle should not be left standing at any place any longer than is absolutely necessary, as it is very apt to leave a streak; therefore, having commenced on a surface, finish it as quickly as possible,—do not let anything interrupt you.

When you get to the bottom dry your brush on a piece of blotting paper, and with the dry brush take up the superfluous tint, as you would with a sponge, until it looks even.

In laying on the tint do not bear on with the brush, as the brush marks would be liable to show, but use the point only, just touching it to the paper so as to wet it, and the tint will follow along of itself (the board being properly inclined).

In following the boundaries of the surface to be tinted let the brush be pointed towards the boundary from the inside of the surface.

This gives what is called a *flat tint*, and is used to represent surfaces which are parallel to the plane of projection.

24. For representing surfaces which are oblique to the plane of projection a graduated tint is necessary. There are two methods of doing this,—the French and the American.

The French method consists in dividing the surface into small divisions (these divisions should be indicated only, not drawn across the surface), laying a flat tint on the first space, and when this is dry laying another flat tint on the first two spaces.

Proceed in this way until the whole surface is covered, commencing at the first space each time. By this method the shading shows streaks of tint of different depths, but are almost unnoticeable if the divisions are taken quite small.

The American method is most used, and is called *shading by softened tints.*

There are two ways of doing this : —

1st. By mixing a small amount of dark wash at first, and starting as if you were to put on a flat tint, and then, by repeated additions of clean water, going over a little more surface at each addition, gradually make the dark tint lighter until you are using almost pure water.

2nd. Divide the surface into divisions, as in the French method, only not so many; put on a flat medium tint on the first space, but, instead of taking up all the tint from the bottom edge of the surface, leave a slight amount, touch the brush to some clean water and apply it to the lower edge of the puddle, thus making a lighter tint, and bring down this new tint a short distance; repeat this a few times until the tint has practically no color. Be careful to remove the most of the tint from the brush each time before touching it to the clear water. This work must be done even quicker than the ordinary flat tint. Use as little tint or water in the brush as you can and not have it dry in streaks. Let this dry, and then repeat the process, commencing at the top and going over two spaces with the flat tint and softening off the lower edge, and so on, commencing at the top each time. Usually the tint should be softened out in the length of one division. If this shading is done perfectly, there will be a gradual change in the tint from beginning to end.

CHAPTER IV.

PROJECTIONS.

25. ORTHOGRAPHIC PROJECTION, or Descriptive Geometry, is the art of representing a definite body in space upon two planes, at right angles with each other, by lines falling perpendicularly to the planes from all the points of the intersection of every two contiguous sides of the body, and from all points of its contour.

26. These planes are called *coördinate planes*, or the *planes of projection*, one of which is *horizontal*, and the other *vertical*. H and V, Fig. 1, represent two such planes and their line of intersection GL is called the *ground line*.

27. We shall only take, in this book, just enough of the elementary principles of projections to enable the student to make *working drawings* of simple objects.

28. Since solids are usually made up of planes, planes of lines, and lines of points, if we thoroughly understand the principles involved in the projections of points, we ought to be able to draw the projections of lines, of planes, and of solids. The only difference being that, with a large number of points, the student is liable to get them confused. To avoid this liability it is advisable, at first, to number the points of a solid and their corresponding projections as fast as found lightly in pencil, and erase them after the problem is finished. Do not try to draw

the object *all at once*, it is impossible; one point at a time is all that can be drawn by anybody, and in this way the most complicated objects become *simple*, even though it may take a long time to complete the drawing.

29. The projection of any point in space on a plane *is the point at which a perpendicular drawn from the given point to the plane pierces the plane.*

This perpendicular is called the *projecting line* of the point. Thus, in Fig. 1, a^h is the projection of the point a on the plane H, and $a_{,}^{v}$ of the same point on the plane V. These are called respectively the *horizontal* and *vertical* projections of the point a; aa^h is called the *horizontal projecting line* of the point a, and $aa_{,}^{v}$ the *vertical projecting line* of the same point.

The horizontal projecting line aa^h is perpendicular to H, by definition, the plane V is assumed perpendicular to H, hence aa^h is parallel to the plane V, and $aa_{,}^{v}$ is equal in length to a^hb. Also the vertical projecting line, for the same reason, is parallel to H, consequently aa^h is equal in length to $a_{,}^{v}b$.

From the definition it is readily seen that each point in a line perpendicular to a plane will have its projection on that plane in one and the same point; hence one projection of a point does not definitely locate its position in space.

30. From the preceding article the following principles may be noted: —

First, the perpendicular distance from the horizontal projection of a point to the ground line is equal to the perpendicular distance of the point in space from the vertical plane; or, briefly, *the horizontal projection of a point indicates the distance of the point in space in front of* V, but it conveys no idea of its distance above H.

Second, the perpendicular distance from the vertical projection of a point to the ground line is equal to the perpendicular

distance of the point in space from the horizontal plane; or, briefly, *the vertical projection of a point indicates the height of the point in space above* H, but it conveys no idea of its distance in front of V.

31. If from the points $a_{,}^{v}$ and a^{h}, Fig. 1, perpendiculars should be erected to each coördinate plane, they will intersect at the point a in space; and as two straight lines can intersect at only one point, there is only one point in space which can have a^{h} and $a_{,}^{v}$ for its projections. Hence *two projections of a point are always necessary to definitely locate its position in space.*

32. It is evident that it would be very awkward to make our drawings on planes at right angles to each other; hence the vertical coördinate plane is supposed to be revolved backward about its line of intersection GL with the horizontal plane until it forms one and the same surface with the horizontal plane, which may be considered to be the plane of the paper.

In this revolution all points in the vertical plane keep the same distance from the ground line, and their relative positions remain unchanged. Thus, $a_{,}^{v}$ revolves to a^{v}, $a^{v}b$ being equal to $a_{,}^{v}b$, and as $a_{,}^{v}b$ was perpendicular to GL before revolution it will be so after revolution, and will form one and the same straight line with $a^{h}b$.

Therefore, *the two projections of a point must always be on one and the same straight line, perpendicular to the ground line.*

Now, if we draw a line across our paper and call it GL, all that portion in front of this line will represent the horizontal plane, and that portion behind it will represent the vertical plane, and the point a located as in Fig. 1 is represented by its projections on the plane of the paper as shown in Fig. 2.

33. *A point situated upon either of the coördinate planes has for its projection on that plane the point itself, and its other projection is in the ground line.*

This is readily seen by referring to Fig. 1; c^v and c^h are the projections of a point on H, and d^v and d^h of a point in V. These points are represented on the plane of the paper as shown by the same letters in Fig. 2.

NOTATION.

34. We will designate a point in space by a small letter, and its projections by the same letter with an h or v written above; thus a^h represents the horizontal and a^v the vertical projection of the point a. This point may be spoken of as the point a, or as the point whose projections are $a^v a^h$.

35. The horizontal coördinate plane will be designated by the capital letter H, the vertical by the capital letter V.

36. Construction lines are those which are made use of simply to obtain required results. They are not a necessary part of the drawing, and when left on a drawing are intended to show the individual steps taken.

To this end the student is expected to ink in all construction lines illustrating the special subject in hand until especially directed not to do so. That is, while on the subject of projections it is not desired to have the geometrical construction lines inked in, but only those which refer to projections. When on the subject of shadows only those which show how the shadow is found are required.

These lines should be inked in with a light, short dash not more than $\frac{1}{16}$ of an inch long, and as light as the student finds he can make easily.

All lines representing the projections of single lines, or edges of planes or solids, if visible, are inked in with a full, continu-

ous line, a little heavier than the construction lines; if invisible, they should be made with short dashes the same length as the construction lines, but the same thickness as the visible lines, so as to distinguish them from the construction lines.

The true length of a line when found should be inked in with a long and short dash, about the same thickness as the ordinary full line. When the true length of a line is given, it is put in like an ordinary construction line.

Indicate an isolated point by drawing a small cross through it.

37. In working drawings — which are practical applications of projections — horizontal projections are usually called *plans*, and vertical projections are called *elevations*. Therefore, they will be used synonymously throughout this book.

38. The student should distinguish between the terms *vertical* and *perpendicular*. *Vertical* is an absolute term, and applies to a line or surface at right angles to the plane of the horizon while *perpendicular* is a relative term, and applies to any line or surface which is at right angles to any other line or surface.

If one point is farther from V than another, the first is said to be in front of the second point. Hence, if I say that a line slopes downward, backward, and to the left, it signifies that the line occupies such a position that the lower end is nearer V than the upper, and also that it is on the left of the upper end. Fig. 6 shows the projections of such a line.

PROJECTIONS OF STRAIGHT LINES.

39. A straight line is determined by two points, therefore it is only necessary to draw the projections of each end of a straight line and join them, and we have the projections of the line. If the line is curved, it becomes necessary to draw the projections of several points and join them with a curve.

40. If we lay a fine wire, ab, Fig. 3, on a horizontal plane, and also parallel to a vertical plane, its horizontal projection will be the wire itself, that is, $a^h b^h$ is its horizontal projection, equal in length to the wire itself, parallel to GL, and at a distance from it equal to the distance of the wire from V.

The vertical projection of the end a will be at a^v in the ground line, and of b at b^v also in the ground line, since each end is on H (Art. 9). Hence the vertical projection of the line will coincide with GL between a^v and b^v, and $a^v b^v$ is its vertical projection. $a^v b^v$ is equal in length to $a^h b^h$, hence is equal to the actual length of the wire.

Now, suppose the wire to be revolved from right to left about a horizontal axis, through the end a, keeping the line parallel to V. If a pencil were attached to the end b at right angles to the line, so that its point touched V at b^v, it would trace a circular arc $b^v c^v d^v$, etc. (of which a^v is the centre and $a^v b^v$, or the true length of the line, is the radius) on the vertical plane as the wire is revolved; the end a would, of course, not move. After the wire has been revolved through an angle of 30° its vertical projection will be at $a^v c^v$, and it must be equal in length to the real length of the wire. a^h will be the horizontal projection of the fixed end of the wire. Since the wire is parallel to V, every point in it is at the same distance from V, hence their horizontal projections must all be the same distance from GL, that is, in a line parallel to GL; the horizontal projection of the end c must also be in a line through c^v perpendicular to GL, hence at c^h where this parallel and perpendicular intersect; $a^h c^h$, then, is the horizontal projection of the wire after it has been revolved through an angle of 30°.

For the same reason $a^v d^v$ and $a^h d^h$ are the two projections of the wire after being revolved through an angle of 45°.

Similarly, $a^v e^v$ and $a^h e^h$ are its projections after revolving through an angle of 60°.

When the line has been revolved through 90° it becomes perpendicular to H, and its vertical projection is $a^v f^v$, perpendicular to GL, and its horizontal projection is a point, a^h, as might have been seen from the definition of the projection of a point.

41. The same reasoning applies if we take the wire lying against V and parallel to H and revolve it about a vertical axis through the end a, as in Fig. 4. The projections are given for the line lying against V, and making angles of 30°, 45°, 60°, and 90° with V, being parallel to H in each position.

42. The following principles may be noted from the preceding articles:—

1*st.* *A line situated in either plane is its own projection on that plane, and its other projection is in the ground line.*

2*nd.* *If a right line is perpendicular to either plane of projection, its projection on that plane will be a point, and its projection on the other plane will be perpendicular to the ground line and equal in length to the given line.*

3*rd.* *When a line is parallel to either coördinate plane its projection on that plane will be parallel to the line itself, and equal to the actual length of the line in space, and its projection on the other plane will be parallel to the ground line.*

4*th.* *If a line is parallel to both planes, or to the ground line, both projections will be parallel to the ground line.*

5*th.* *If a line is oblique to either coördinate plane its projection on that plane will be shorter than the actual length of the line itself.*

6*th.* *If a line is parallel to one coördinate plane and oblique to the other, its projection on the plane to which it is parallel is equal to the true length of the line in space, and the angle which this projection makes with the ground line is equal to the true size of the angle the line in space makes with the plane to which it is oblique.*

7th. The projection of a line on a plane can never be longer than the line itself.

8th. If a point be on a line its projections will be on the projections of the line.

9th. If a line in different positions makes a constant angle with a plane its projections on that plane will all be of the same length, without regard to the position it may occupy relative to the other plane.

43. If two lines intersect in space their projections must also intersect, and the straight line joining the points in which the projections intersect must be perpendicular to the ground line; for the intersection of two lines must be a point common to both lines, whose projections must be on the horizontal and vertical projections of each of the lines, hence at their intersections respectively.

44. If two lines are parallel in space their projections upon the vertical and horizontal planes will be parallel respectively. If one projection only of two lines are parallel, the lines in space are not parallel.

45. Any two lines drawn at pleasure, except parallel to each other and perpendicular to the ground line, will represent the projections of a line in space.

46. Prob. 1. *To draw the projections of a line of a definite length and occupying a fixed position in space.*

Let it be required to draw the projections of a line 1″ long, which makes an angle of 30° with H, and whose horizontal projection makes an angle of 45° with GL, the lower end of the line being $\frac{1}{2}''$ above H and $\frac{1}{4}''$ in front of V.

It is first necessary to place the line in such a position that its true length and the true size of the angle it makes with one of the coördinate planes are shown, and these are only shown

when it is parallel to one of the coördinate planes. In this case it must be placed parallel to V. a^v and a^h, Fig. 5, are the projections of one end of the line, a^v being $\frac{1}{2}''$ above GL and a^h $\frac{1}{4}''$ below it. Through a^v draw $a^v b_{\prime}^v$ at an angle of 30° with GL and 1″ long; through a^h draw $a^h b_{\prime}^h$ parallel to GL, $b_{\prime}^h$ being found by dropping a perpendicular from b^v (Art. 40). The two projections of the line, when parallel to V, are thus found to be $a^v b_{\prime}^v$ and $a^h b_{\prime}^h$.

Now let us suppose the end a of the line to be fixed and the whole line to be revolved through an angle of 45° about a vertical axis through this point, the line keeping the same angle with H. The horizontal projection will not change in length (Art. 42, 9th), but will move through an angle of 45°, and will be found at $a^h b^h$. It is evident that in this revolution, so long as the angle with H does not change, every point in the line will remain at the same height above H. The point a does not move, being in the axis. We have seen that $b_{\prime}^h$ moves to b^h; b^v must, therefore, be somewhere on a perpendicular through b^h, and, since the points do not change their heights, it must also be on a line through $b_{\prime}^v$ parallel to GL, hence at their intersection b^v. Join a^v and b^v and we have $a^v b^v$ and $a^h b^h$ as the required projections of the line.

47. If this line were revolved through 15° more, the point b^h would go to c^h, and b^v to c^v, and $a^v c^v$ and $a^h c^h$ would be the projections of the line making an angle of 30° with H, and whose horizontal projection made an angle of 60° with GL.

If it were revolved still 30° more, the two projections would be $a^v d^v$ and $a^h d^h$, each being perpendicular to the ground line. When a line is in this position, i. e., has its two projections in a line perpendicular to GL, it is said to be in a *profile* plane, a profile plane being understood to be one that is perpendicular to both V and H.

When a line is oblique to only one of the coördinate planes it is said to make a *simple* angle; when it is oblique to both of them it is said to make a *compound* angle.

48. If the angle that the line made with V had been given, it would have been necessary to have first placed the line parallel to H, and then to have revolved it about an axis through one end perpendicular to V, in which case the length of the vertical projection would not change, and the points would not change their distances from V.

In Fig. 6, $a^v b^v$ and $a^h b^h$ are the two projections of a line 1″ long, making an angle of 45° with V, and whose vertical projection makes an angle of 60° with GL. The principles and explanation for this construction are the same as for Prob. 1, if the horizontal and vertical planes are supposed to be interchanged.

49. PROB. 2. *To find the true length of a line given by its projections, and the angle it makes with either plane of projection.*

Let $a^v b^v$ and $a^h b^h$, Fig. 7, be the projections of the given line. The true length is only shown when it is parallel to one of the coördinate planes, hence this line must be revolved about an axis through either end until it is parallel to one of the planes. If it is revolved about a vertical axis through a until it is parallel to V, the point a does not move, b^h moves to $b_{\prime}^{h}$, b^v is found at $b_{\prime}^{v}$ (where a perpendicular through $b_{\prime}^{h}$ intersects a horizontal through b^v), and $a^v b_{\prime}^{v}$ is the true length. Also, the angle which $a^v b_{\prime}^{v}$ makes with GL is equal to the true size of the angle the line makes with H.

If it had been required to find the angle this line made with V, it would have been necessary to have revolved the line about a horizontal axis until it became parallel to H. Assuming the

axis through the end b, Fig. 7, a^v moves to $a_{,}^{v}$, a^h to $a_{,}^{h}$, and $a_{,}^{h}b^h$ is the true length of the line (which of course should equal $a^v b_{,}^{v}$), and the angle it makes with GL is equal to the true size of the angle the line makes with V.

50. NOTE. *The angles which the vertical and horizontal projections of a line make with* GL *are greater than the angles which the line in space makes with* H *and* V *respectively, except when the line is parallel to one of the planes.*

PROJECTIONS OF SURFACES.

51. Plane surfaces are bounded by lines, therefore the principles which govern the projections of lines are equally applicable to these surfaces.

52. If we suppose a rectangular card *abcd*, Fig. 8, placed with its surface parallel to V and perpendicular to H, each edge being parallel to V, it will be projected on V in a line equal and parallel to itself, hence the true size of the card itself is shown in vertical projection. Two of the edges, *ab* and *cd*, being perpendicular to H, are projected on that plane in the points a^h and d^h respectively. The other two edges, *ad* and *bc*, are parallel to H as well as V, hence they will be projected in their true length on H, and, since one is vertically over the other, they will both be horizontally projected in the same line $a^h d^h$.

Now, if the card is revolved about one of its vertical edges as an axis, like a door on its hinges, the vertical edge which coincides with the axis does not move; the other vertical edge moves in the arc of a circle. The horizontal projection of the card will still be a straight line of the same length as before. Let the card be revolved through 60°; a^h does not move; d^h moves in the arc of a circle, of which a^h is the centre and $a^h d^h$ the radius. to $d_{,}^{h}$; $a^h d_{,}^{h}$ is the horizontal projection of the card in its new position; the vertical projection of the edge *cd* in

this position is found at $c_{\prime}^{v}d_{\prime}^{v}$, vertically above $d_{\prime}^{h}$, and $a^{v}b^{v}c_{\prime}^{v}d_{\prime}^{v}$ is the vertical projection of the card after being revolved through an angle of 60°.

If the card should be revolved through 30° more, i. e., 90° in all, its surface will be at right angles with both coördinate planes, and its two projections will be found at $a^{v}b^{v}$ and $a^{h}e^{h}$, in one and the same straight line perpendicular to GL.

53. If the card be placed on H, with one of its edges parallel to V, $a^{h}b^{h}c^{h}d^{h}$, Fig. 9, will be its horizontal and $a^{v}b^{v}$ its vertical projection. If this card be revolved about one of its edges which are perpendicular to V as an axis, like a trap-door on its hinges, through an angle of 30°, $a^{v}b_{\prime}^{v}$ and $a^{h}b_{\prime}^{h}c_{\prime}^{h}d^{h}$ will be its two projections. If it be revolved through 60° more, or 90° in all, its projections will be $a^{v}e^{v}$ and $a^{h}d^{h}$, which are just the same as $a^{v}b^{v}$ and $a^{h}e^{h}$ in Fig. 8, as they should be, since the cards are the same size in the two figures and they occupy the same relative position in each, i. e., they are in a profile plane.

54. The following principles may be noted: —

1*st.* *When a plane surface is perpendicular to another plane its projection on that plane will be a line.*

2*nd.* *When a plane surface is parallel to either coördinate plane, its projection on that plane will be equal to the true size of the surface and its other projection will be a line parallel to* GL.

3*rd.* *When a plane surface is perpendicular to one plane and oblique to the other, the angle which its projection on the plane to which it is perpendicular makes with the ground line is equal to the angle the surface in space makes with the plane to which it is oblique.*

4*th.* *If a plane surface, in different positions, makes a constant angle with a plane, its projections on that plane will all be of the same size.*

55. PROB. 3. *To draw the two projections of a plane surface, or card, of a certain size, and making a compound angle with the coördinate planes.*

Let the card be of the size shown in Fig. 9, and suppose it to make an angle of 30° with H, and its horizontal projection an angle of 45° with GL.

Draw the horizontal projection $a^h b^h c^h d^h$ of the card equal to its true size; $a^v b^v$ will be its vertical projection. Revolve $a^v b^v$ through an angle of 30° to $a^v b_{,}^v$, and $a^v b_{,}^v$ will be the vertical projection of the card when it makes an angle of 30° with H and is perpendicular to V; $a^h b_{,}^h c_{,}^h d^h$ is its corresponding horizontal projection.

The angle with H is still to be 30° after the card has been revolved to its desired position, hence its horizontal projection will be the same size. Therefore, make $a^h b_{,}^h c_{,}^h d^h$, Fig. 10, equal in size to $a^h b_{,}^h c_{,}^h d^h$, Fig. 9, and making the desired angle with GL. In this revolution, as long as one edge rests on H and the angle remains constant with H, every point keeps the same height above H, therefore the vertical projections of a^h and d^h, Fig. 10, must be found at a^v and d^v; also of $b_{,}^h$ and $c_{,}^h$ at $b_{,}^v$ and $c_{,}^v$, whose heights above GL are equal to the height of $b_{,}^v$, Fig. 9, above GL.

The other parallelograms in Fig. 10 represent the projections of the same card at different angles with H, the horizontal projections making the same angles with GL.

56. $a^h b^h c^h d^h$ and $a^v d^v b^v c^v$, Fig. 11, represent the projections of the same card when it is lying on H with one of its diagonals parallel to V, and $a^v d_{,}^v b_{,}^v c_{,}^v$ and $a^h b_{,}^h c_{,}^h d_{,}^h$ are its projections after being revolved about an axis perpendicular to V through the corner a through an angle of 45°. Fig. 12 represents the projections of this card when, besides making an angle of 45° with H, the horizontal projection of the diagonal makes

an angle of 30° with GL. The steps to obtain this are exactly the same as in Figs. 9 and 10, hence the explanation will not be repeated.

57. PROB. 4. *To draw the projections of a regular pentagonal card, the diameter of the circumscribed circle being given, in two positions. 1st, when it is perpendicular to* V *and making an angle of* 60° *with* H, *one of its edges being perpendicular to* V; *2nd, when, besides making an angle of* 60° *with* H *as in 1st position, it has been revolved through an angle of* 45°.

The pentagon must first be drawn in its true size and position; $a^h b_{\prime}^h c_{\prime}^h d_{\prime}^h e_{\prime}^h$, Fig. 13, equal to the actual size of the card, is its horizontal projection, $c_{\prime}^h d_{\prime}^h$ being perpendicular to GL, and $a^v b_{\prime}^v c_{\prime}^v$ is its vertical projection. Revolve $a^v b_{\prime}^v c_{\prime}^v$ through an angle of 60° to $a^v b^v c^v$; each point moves in the arc of a circle with a as a centre, and $a^v b^v c^v$ will be the vertical projection of the card when in the 1st position asked for; $a^h b^h c^h d^h e^h$ is its corresponding horizontal projection.

For the 2nd position revolve the plan just found through 45° to the position $a^h b^h$, etc., shown in Fig. 14; $a^v b^v c^v d^v e^v$ will be the corresponding vertical projection.

58. Cards of any shape and size, and occupying any position, may be drawn in the same way, care being taken to locate one point at a time.

59. If the angle the card made with V had been given, it would have been necessary to have first placed the card parallel to V and then to have revolved it, through the angle it made with V, about an axis perpendicular to H, in which case the length of the horizontal projection, which is a straight line, would not change, and the several points would not change their respective distances from H.

In the second revolution, which changes the angle with the coördinate planes from *simple* to *compound*, the vertical projection must be revolved and the corresponding plan found. The angle with V being the same the vertical projection does not change its size; the distances of the points in front of V remain the same after revolution as before, hence are found at the same distances from GL respectively.

60. PROB. 5. *To draw the projections of a circular card making a compound angle with the coördinate planes.*

Let the diameter of the card be given, the angle it makes with V, and the angle through which the vertical projection is to be revolved.

A circle may be considered as a polygon of an infinite number of sides, hence we can take as many points as we please on the circumference of the circle, and each one moves according to the principles just described.

Place the card parallel to V; a circle, $a^v b_{,}^v c_{,}^v$, etc., Fig. 15, equal to the actual size of the given circle, is its vertical projection, and $a^h b_{,}^h c_{,}^h$, etc. is its horizontal projection.

Revolve the card through the required angle about a vertical axis through a; $a^h b^h c^h$, etc. is its horizontal, and $a^v b^v c^v$, etc. is its vertical projection.

The card is to be revolved through a certain angle, still keeping the same angle with V. The *size* of the vertical projection will, therefore, not change. Hence, revolve the vertical projection found in Fig. 15 through the required angle to the position $a^v b^v c^v$, etc., Fig. 16. None of the points change their distance from V, consequently $a^h b^h c^h$, etc. is the horizontal projection of the card, found as in the last problem.

61. Fig. 17 shows a somewhat shorter method of drawing the projections of a circular card making a simple angle with

the coördinate planes; in this case it makes an angle of 30° with V and is perpendicular to H. $a^hb^hc^h$, etc., making 30° with GL, is its horizontal projection. Suppose the card revolved about its horizontal diameter ae until it is parallel to H. It will then be shown in its true size at $a^hb_{,}c_{,}$, etc.; $b_{,}b^h$, $c_{,}c^h$, etc. will show the actual distances of the points b, c, etc. from the horizontal diameter; a^ve^v will represent the vertical projection of the horizontal diameter about which the card is revolved. Of course, b^v must be found somewhere in a line through b^h, perpendicular to GL, therefore, lay off tb^v equal to $b_{,}b^h$, and b^v is a point of the required vertical projection; c^vs is made equal to $c_{,}c^h$, d^vr to $d_{,}d^h$, etc. Other points may be found in the same way.

It is evident that $n_{,}b^h$, $m_{,}c^h$, etc. are respectively equal to $b_{,}b^h$, $c_{,}c^h$, etc., hence it is only necessary to revolve the *semicircle* and the distance $b_{,}b^h$ is laid off on both sides of the diameter a^ve^v, giving the two points b^v and n^v.

PROJECTIONS OF SOLIDS.

62. A cube is a solid bounded by six equal faces, and when it is placed so that two of its faces are parallel to H, and two others parallel to V, its two projections are $a^vb^ve^vf^v$ and $a^hb^hc^hd^h$, Fig. 18. The top and bottom, being parallel to H, are horizontally projected in one and the same square, $a^hb^hc^hd^h$, which is, of course, equal to the exact size of these faces, and their vertical projections are a^vb^v and e^vf^v respectively (Art. 30-2nd); the front and back faces being parallel to V are vertically projected in one and the same square, $a^vb^ve^vf^v$, which is also equal to the exact size of these faces, and their horizontal projections are a^hb^h and c^hd^h respectively (Art. 30-2nd); the left and right hand faces are perpendicular to both V and H, therefore their vertical projections are a^ve^v and b^vf^v, and their horizontal projections are a^hd^h and b^hc^h respectively (Art. 30-1st).

The plan shows two dimensions of the cube, the length and breadth, and the elevation two dimensions, the length and thickness; therefore, the three dimensions of the solid being shown in their true size in the two projections the object is completely represented. In this case it does not matter which projection is drawn first, as they each show two dimensions in their true size.

63. SHADE LINES. In outline drawings it is customary to put in shade lines, i. e., lines heavier than the others; they give relief to the drawing, and, *when properly placed*, are of assistance in reading it.

Shade lines, or edges, are those edges which separate light from dark surfaces.

The rays of light are generally assumed to come from over the left shoulder in the direction of the diagonal of a cube, the person supposed to be facing the cube, and the cube to be in the position shown in Fig. 18. That is, the ray of light enters the cube at the upper, front, left-hand corner, whose projections are a^v and a^h, and leaves it at the lower, back, right-hand corner, whose projections are f^v and c^h; the diagonal joining these points will represent the actual direction of the conventional ray of light, and its projections a^vf^v and a^hc^h are the projections of this ray. The different rays of light are all supposed to be parallel to each other.

It is evident from the figure that both projections of the rays of light make angles of 45° with GL. The student should distinctly understand that, although the projections of the ray of light make 45° with GL, the actual angle it makes with V or H is quite different. To find this angle apply the principles of Art. 25 to Fig. 18, and we get $\alpha = 35° \ 15' \ 52''$ as its actual size.

64. In the cube, Fig. 18, the top, front, and left-hand faces

are light, and the bottom, back, and right-hand faces are dark, and the shade lines are, therefore, e^vf^v, which separates the front from the bottom, b^vf^v, which separates the front from the right-hand face, b^hc^h, which separates the top from the right-hand face, and c^hd^h, which separates the top from the back face. The two other shade edges which separate the left-hand face from the back and bottom faces are not seen in either projection, since the top and front edges of the left-hand face are in the same plane and nearer the eye.

It is for this same reason that the shade lines mentioned above are seen only in one projection, i. e., e^vf^v is a shade line; it is seen in elevation, but in plan is hidden by the upper front edge of the cube a^vb^v–a^hb^h.

65. It will be noticed that the right-hand and lower edges are shaded in elevation, and the right hand and upper in plan. From this many draftsmen have adopted the arbitrary rule to shade the right-hand and lower lines in elevation, and the right-hand and upper in plan. This rule is really applicable in but few cases, except when the object is of rectangular section and so placed that its surfaces are perpendicular to one or both of the coördinate planes.

Other draftsmen shade the right-hand and lower lines in both plan and elevation. This is also applicable only in the cases stated above, besides being obliged to change the direction of the ray of light in plan and elevation, or imagine the object revolved while the ray of light remains fixed.

Others still follow the rule given in Art. 63, except that they only put in shade lines where the dark portion of space adjacent to the line in question is visible, and they change the direction of the ray of light, or, what is practically the same thing, revolve the object. By this method the plan and elevation are shaded alike, but the right-hand line is not necessarily a shade line, while the left-hand line is necessarily not a shade line.

The method taken up in this book, and the one it is expected that the student will follow in this course, is not given because it is the one most generally in use, or because it is the easiest,—quite the contrary; but because it is, in the opinion of the writer, the *only* method which can be followed consistently throughout a course of projections, shadows, isometric, and working drawings.

66. PROB. 6. *To draw the two projections of a right square prism with its base on* H *and its vertical faces oblique to* V. Fig. 19.

It is evident that the vertical faces will not be projected on V in their true size, the height only of the prism being shown in its real length, and also that the two ends, being parallel to H, will be projected on that plane in their true size. Hence, draw the square $a^hb^hc^hd^h$ equal to the ends of the prism, with its edges making the same angles with GL as the vertical faces make in space with V. Through the corners a^h, b^h, c^h, and d^h, draw the perpendiculars a^ve^v, b^vf^v, etc., making each equal in length to the height of the prism. $a^vb^vc^vd^v$ will be the vertical projection of the upper base, $e^vf^vm^vn^v$ of the lower base, and $a^vc^vm^ve^v$ the vertical projection of the whole prism.

In this case the top and two front faces are light, while the bottom and two back faces are dark; hence, in plan the lines a^hd^h and c^hd^h, which separate the top from the two back faces, are shade lines; in elevation they are behind the two front edges ab and bc, consequently are not seen. The element ae separates the left front from the left back face, hence is a shade line, and a^ve^v, its vertical projection, is accordingly made heavy; its horizontal projection is simply a point; the element cm also separates a light from a dark surface, and its visible projection c^vm^v should be made heavy. The edges ef and fm separate the two

front faces from the bottom, and their visible projections $e^v f^v$ and $f^v m^v$ are also made heavy.

67. Prob. 7. *To draw the two projections of a right regular pentagonal prism standing with its base on* H, *with none of its faces parallel to* V. Fig. 20.

Here, as in the last problem, it is necessary to draw the horizontal projection first, as it shows the pentagonal end in its true size and position. $a^h b^h c^h d^h e^h$ is its horizontal projection, and $a^v d^v n^v f^v$, found as in the last problem, is its vertical projection.

From the shade lines shown in the figure it will be noticed that the line *cd*, which separates the light top from the dark right-hand face, is visible in both projections, hence it is made heavy in both.

A prism of any number of sides standing on H can be drawn in the same manner.

68. Prob. 8. *To draw the two projections of a right regular hexagonal prism with its axis perpendicular to* V. Fig. 21.

Here it is necessary to draw the vertical projection first, and construct the horizontal projection from it according to the principles noted in the last two problems.

69. From an inspection of Figures 18 to 21 it is evident that *the 45° triangle can be used to determine positively the light and dark faces only when these faces are perpendicular, or nearly so, to one or both of the coördinate planes.*

In Fig. 18 the triangle can be used in both plan and elevation, since every face is perpendicular to at least one of the coördinate planes. In Figs. 19 and 20 the faces are perpendicular to H only (except the top, which is, of course, known to be light), hence the 45° triangle can only be used in the plan.

Fig. 1

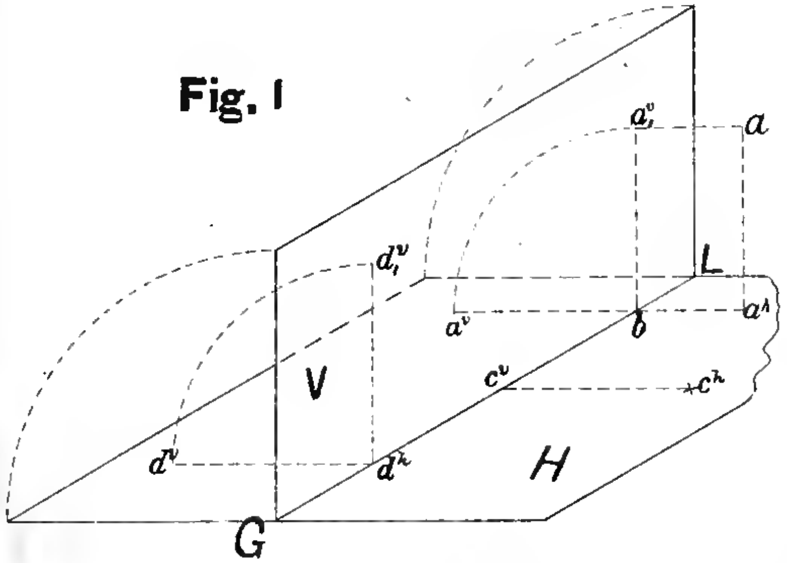


Fig. 2

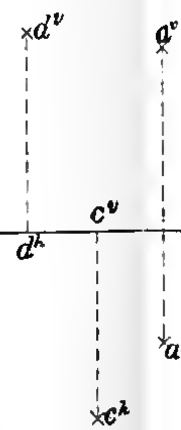


Fig. 3

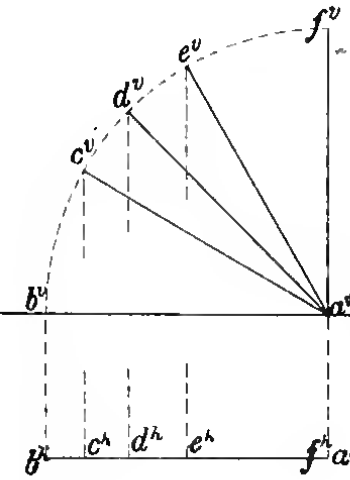


Fig. 4

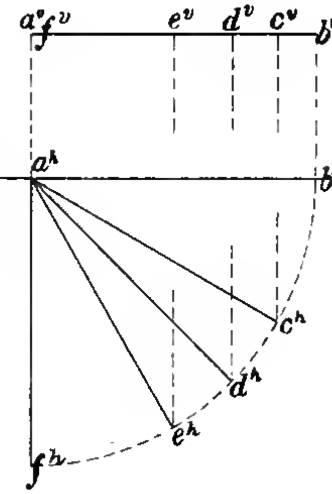


Fig. 5

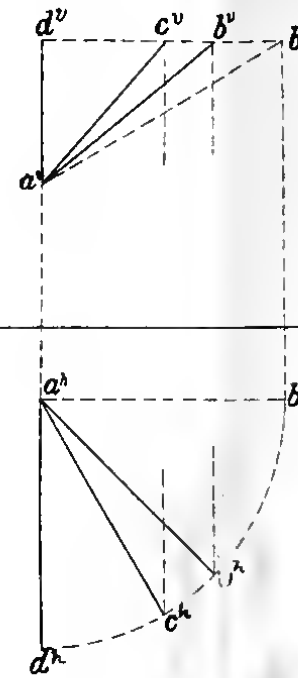


Fig. 6

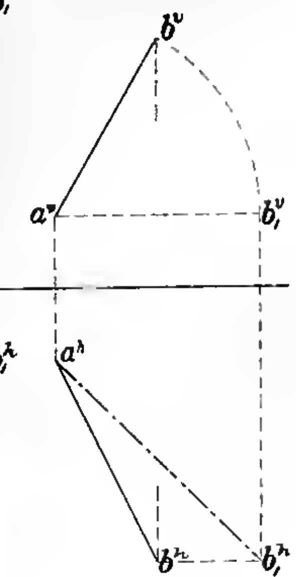


Fig. 7

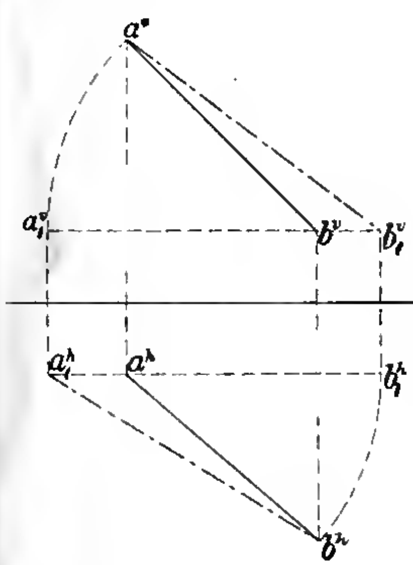


Fig. 8

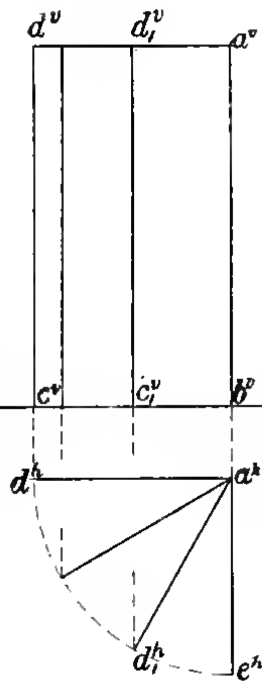


Fig. 9

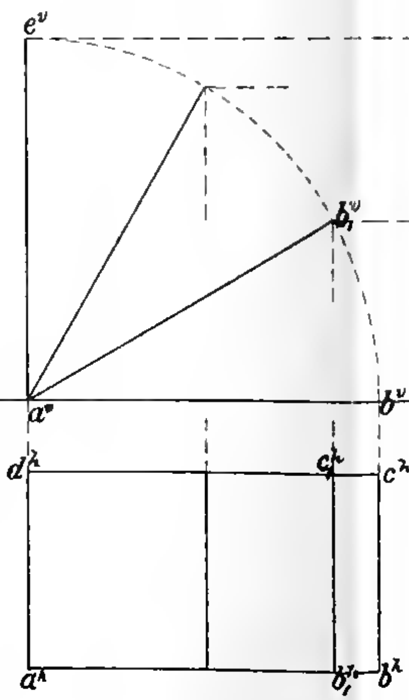


Fig. 10

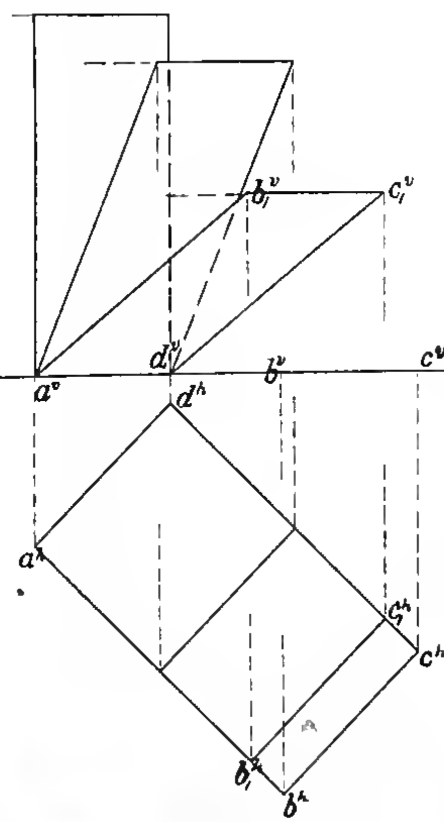


Fig. 11

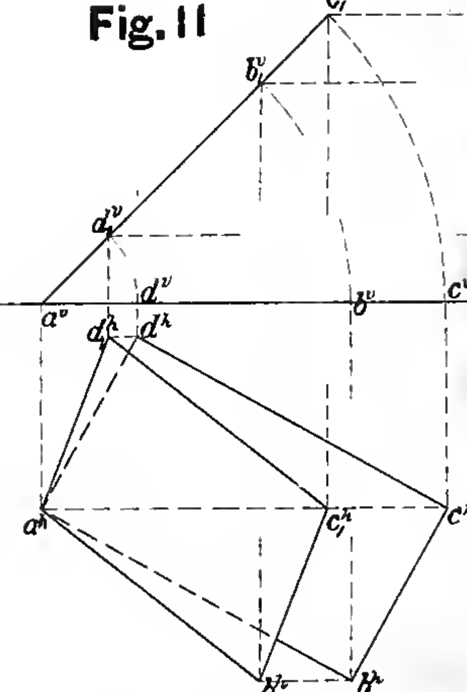
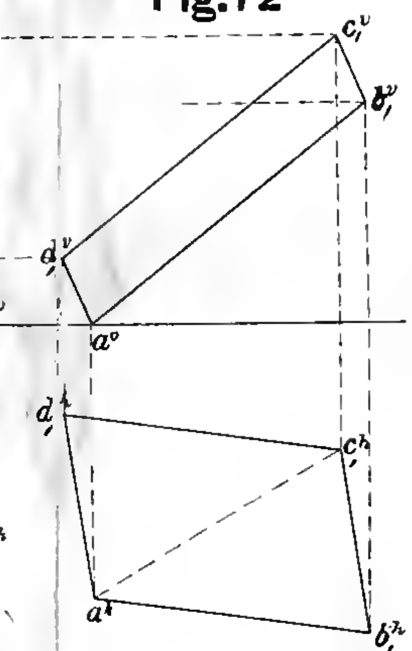


Fig. 12



In Fig. 21 the faces are perpendicular to V only (except the front end, which must be light), hence the triangle can only be used in elevation.

70. Prob. 9. *To draw the two projections of a right cylinder standing on its base.* Fig. 22.

Its horizontal projection will be a circle equal to the end of the cylinder (Art. 42). Its vertical projection will be represented by $a^v b^v d^v c^v$, $a^v c^v$ being the vertical projection of the top, $b^v d^v$ of the bottom, $a^v b^v$ and $c^v d^v$ of the extreme left and right hand elements, or the contour lines as they are called.

By applying the 45° triangle it is evident that the shade line in plan will be the half circle $m^h c^h o^h$ between the points where the triangle is tangent to the circle. The element *cd* is not a shade line, as it does not separate a light from a dark surface. The shade element would be *or*, but as it is not drawn of course it cannot be shaded.

The bottom of the cylinder is dark, and *strictly* the line $b^v r^v$ would be heavy, leaving the portion $r^v d^v$ as light; but, since it is practically impossible to stop the shade line at the point r^v and make a good-looking line, I should disregard this short piece and shade the line the whole length. Similarly, on top there will be a short piece between o^v and c^v that would strictly be shaded, but for the same reason I would disregard this and make the whole top a light line.

The absence of the shade line $c^v d^v$ in the vertical projection enables us to tell at once that the object is cylindrical even before we look at its plan.

A cone is like a cylinder, except that its elements all intersect in a common point called the vertex, while in the cylinder they are all parallel. Fig. 51 represents the two projections of a cone.

71. Prob. 10. *To draw the two projections of a right regular hexagonal pyramid with its base resting on* H. Fig. 23.

As we look directly down upon a pyramid, in this position we shall see all of its sloping faces, and consequently the edges which separate these faces. We shall also see the edges which separate these sloping faces from the base, i. e., the outline of the base. In this case the base is parallel to H, hence its outline is shown in plan equal to the real size of the base. Therefore, the regular hexagon, $a^hb^hc^hd^he^hf^h$, is the horizontal projection of the outline of the base. Now, since in a right pyramid the base is perpendicular to the axis, it will be easily seen that the horizontal projection of the vertex of the pyramid must be at o^h, the centre of the hexagon.

Drawing lines from o^h to each corner of the hexagon, the horizontal projection of the pyramid is completed. The points in the base are, of course, vertically projected in GL, the vertex at o^v at a distance above GL equal to the altitude of the pyramid. Joining o^v with a^v,b^v,c^v, etc. we have its vertical projection.

The shade lines of a pyramid are not found directly by means of the 45° triangle, as we have been able to do previous to this, on account of the faces not being perpendicular to either coördinate plane. If we try to use the triangle as in the case of the prism, we would have said that the three faces, $f^ho^ha^h$, $a^ho^hb^h$, and $b^ho^hc^h$ were light, and the three remaining faces dark, but this is not the case. For let us suppose that the altitude of this pyramid is so small that each of the faces of the pyramid makes an angle with H less than 35° 16′ (the angle the ray of light makes with H). It is evident that *all* of the sloping faces will be light, and the bottom being dark the shade lines would go entirely round the base. Now, if we consider the altitude to increase, we shall soon reach the point when the

face $o^h d^h e^h$ will become dark, all of the rest remaining light, and the shade line would change from $d^h e^h$ to $e^h o^h$ and $o^h d^h$. If the altitude be still further increased, we next get the case shown in the figure where the face $f^h o^h e^h$ becomes dark, and the shade lines would change from $e^h f^h$ and $e^h o^h$ to $f^h o^h$. If the altitude should be still further increased, the face $c^h o^h d^h$ would presently become dark also.

Of course the other three faces would never become dark while the pyramid rested on its base, even if the vertex were extended to infinity, in which case we should simply have a prism. In cases like this, or where any surface is oblique to both V and H, it is necessary to find the shadow of the object, thus determining which surfaces are light and which are dark.

72. In Fig. 24 $a^v b^v f^v e^v$ is the elevation and $a^h b^h c^h d^h$ is the plan of a rectangular prism, with two of its faces parallel to each of the coördinate planes. The plan shows its length and width, and the elevation its length and thickness. If a side elevation is desired, it will show the width and thickness. To get this the object must be projected onto a plane at right angles to the two coördinate planes, i. e., the profile plane, and this plane revolved about its intersection with V, as an axis, to coincide with V. POR is such a plane, resting against the end of the prism, PO being its intersection with V and OR its intersection with H.

In this revolution none of the points change their heights above H, nor their distances from the axis PO, hence the rectangle $b_{,}^{v} c_{,}^{v} f_{,}^{v} m_{,}^{v}$ will represent this side elevation, it being of course the same height above GL that the front elevation is, and the distance that $c_{,}^{v} m_{,}^{v}$ is from the axis PO will be equal to the distance the back of the prism is in front of V.

The shade lines in the end elevation are shaded the same way

as in the front elevation; the ray of light is supposed to come from over the person's left shoulder when he is facing the profile plane, i. e., *the vertical projection of the ray of light is the same for all elevations.*

73. PROB. 11. *To draw the plan and two elevations of a square prism with its axis parallel to and at a definite distance from both* V *and* H, *all of its faces being oblique to both* V *and* H. Fig. 25.

The end elevation is the only view of the prism which shows one of its surfaces in its true size and position relative to the coördinate planes, hence this view must be drawn first.

Locate the point o^v at a perpendicular distance above GL equal to the height of the axis above H; through o^v draw two lines at right angles to each other, making angles with GL equal to those made by the long faces of the prism with H respectively; lay off on each of these lines, on both sides of o^v, a distance equal to half the side of the square; through these points draw lines parallel to the lines through o^v, and the square $a_{,}^{v}b_{,}^{v}c_{,}^{v}$ $d_{,}^{v}$ thus formed will be the end elevation of the prism in its correct position.

To locate the axis the correct distance from V draw POR, which represents the profile plane, perpendicular to GL and at a horizontal distance to the right of o^v, equal to the distance of the axis in front of V.

In the last problem the end view was constructed from the plan and front elevation; in this problem we construct the plan and front elevation from the end view by simply reversing the steps.

The horizontal lines, a^ve^v, b^vf^v, c^vm^v, and d^vn^v, drawn through $a_{,}^{v}$, $b_{,}^{v}$, $c_{,}^{v}$, and $d_{,}^{v}$ respectively, and each equal in length to the length of the prism, will be the front elevation of the different

elements of the prism; joining these ends the front elevation of the prism is complete.

From O along OR lay off Om^h, Of^h, etc. equal to the horizontal distances of $c_{,}^{v}$, $b_{,}^{v}$, etc. from PO. Through these points m^h, f^h, etc. draw the horizontal lines m^hc^h, f^hb^h, etc., each equal in length to the length of the prism, and joining the ends the plan of the prism, $a^hc^hm^he^h$, will be complete.

The long faces of the prism being perpendicular to V in the end view, the shade lines for that view may be found directly by using the 45° triangle, as shown by the arrows. In revolving the prism from the position shown in end view to that in the front view, the front and back ends change from light to dark and from dark to light respectively, but the long faces are light or dark in the front view and plan according as they are light or dark in the end view (provided the projection of the right-hand end is represented, which will be seen to the left of the front view).

74. Prob. 12. *To draw the two projections of a regular pentagonal prism, with its axis parallel to* H *and oblique to* V, *and its lower left-hand long face making a definite angle with* H. Fig. 26.

Here, as in the last problem, it is necessary to draw the view of the end of the prism when its axis is perpendicular to V, so as to show it in its true size and position. $a_{,}^{v}b_{,}^{v}c_{,}^{v}d_{,}^{v}e_{,}^{v}$ is its end view, the edge $a_{,}^{v}b_{,}^{v}$ making an angle with GL equal to the angle the lower left face makes with H. In the last problem the prism was revolved through an angle of 90° to its actual position, but in this it is revolved through a smaller angle. The steps being otherwise just the same, the explanation will not be repeated.

The shade lines in this case may also be found, as in the last problem, by using the 45° triangle on the end view.

75. Let us now suppose a case where the edge $c_{\prime}^{v}d_{\prime}^{v}$, Fig. 26, makes an angle of 40° with H and in the same direction. It is evident that when the axis of the prism is perpendicular to V the surface which is projected in $c_{\prime}^{v}d_{\prime}^{v}$ will be light. Now, if the prism be revolved through 45° so that its axis makes an angle of 45° with V, in the same direction as shown in Fig. 26, it is also evident that the surface, which makes an angle of 40° with H, will now be dark, and the shade lines would therefore change. If the prism be revolved through 45° more in the same direction, its axis would be parallel to V, and the surface in question would then be light. That is, the surface when perpendicular to V would be light, but as it was revolved parallel to H, at some intermediate position before it had revolved 45°, it would become dark, changing to light again at some intermediate position between 45° and 90° of revolution.

The same thing would occur if the face in question made any angle with H between 35° 16′ and 45°. The foregoing reasoning would apply equally well to the under face $a_{\prime}^{v}b_{\prime}^{v}$, except that this one would be dark where the corresponding upper one would be light.

Therefore, if a surface of a prism, as in the last problem, makes an angle with H *between* 35° 16′ *and* 45°, *that surface becomes doubtful in all its positions when the axis of the prism is oblique to* V, *and the shadow of this surface would have to be cast to determine positively whether it is light or dark.*

If the surfaces make angles with H, not included between the above limits, the 45° triangle on the end view would determine the light and dark surfaces for all the oblique positions of the prism, as well as when the axis is perpendicular to V or parallel to V and H.

76. Fig. 27 represents the two elevations and plan of a hollow cylinder whose axis is parallel to V and H. Here the end

elevation would naturally be drawn first, as in the last two problems, but it is not strictly necessary, as both of its projections, when parallel to V and H, are the same, and the distance apart of the contour elements is equal to the diameter of the base.

The student should note carefully the shade lines in the figure, especially in the end view.

77. Fig. 28 shows the plan and two elevations of a pile of blocks. The lower one is a rectangular prism, the second one, which rests on the first, is the frustum of a square pyramid, and the top one is a square pyramid. In this case it is necessary to draw the plan first and construct the two elevations from it, according to the principles already explained.

It will be observed that the group is considered as solid in putting in shade lines, i. e., the edges which represent the perimeter of the base of the pyramid, for example, are considered as separating the sloping faces of the pyramid from the top surface of the frustum on which it rests, and not from its base, as in Fig. 23. Compare the shade lines of the pyramids in Figs. 23 and 28.

Since it is customary to tint drawings in which shadows are cast, shade lines would not be put in on the same drawing. Therefore, it is advisable to disregard the shadows altogether in putting in shade lines on line drawings.

78. PROB. 13. *To construct the projections of a right hexagonal prism when its axis makes a compound angle with the coördinate plane, the angle it makes with* H, *the angle the horizontal projection makes with* GL, *and the size of the prism being given.* Figs. 29, 30, and 31.

There are three distinct steps necessary in the construction of this problem.

Since the axis is oblique to both planes the prism cannot be

drawn in the required position directly, but must be placed in such a position as will show two dimensions. Here the first step is to draw the two projections of the prism when its axis is perpendicular to H and its faces make the required angles with V (Art. 67). Fig. 29 represents the projections of the prism in this position.

Next draw the projections of the prism after it has been revolved in the proper direction, so that the axis makes the correct angle with H, but is still parallel to V. Fig. 30 represents the projections of the prism in this position. Since the prism keeps a constant angle with V, its vertical projection does not change its size (Arts. 42–9th and 54–4th). Hence, make the vertical projection in Fig. 30 the same size as in Fig. 29, with its position changed so that the elements make the correct angle with GL. In this revolution no point changes its distance from V. Therefore, to construct the horizontal projection of the prism in this second position, draw through each corner, as a^v, $a_{\prime}^{v}$, etc., Fig. 30, lines perpendicular to GL until they intersect horizontal lines drawn through the corresponding points, as a^h in Fig. 29. This completes the second step.

In the final position the axis is to make the same angle with H as it does in the position just drawn. Hence the horizontal projection must be the same size, however much it may be revolved. In this revolution no point changes its height above H. Therefore, draw the plan of the prism, Fig. 31, the same size as that in Fig. 30, only changing the angle the elements make with GL the required amount. Then from each corner of this plan, as a^h,$a_{\prime}^{h}$, etc., draw perpendiculars to GL until they intersect horizontal lines drawn through the corresponding points, a^v,$a_{\prime}^{v}$, etc., in Fig. 30. Joining these corners the vertical projection is completed.

Fig. 13

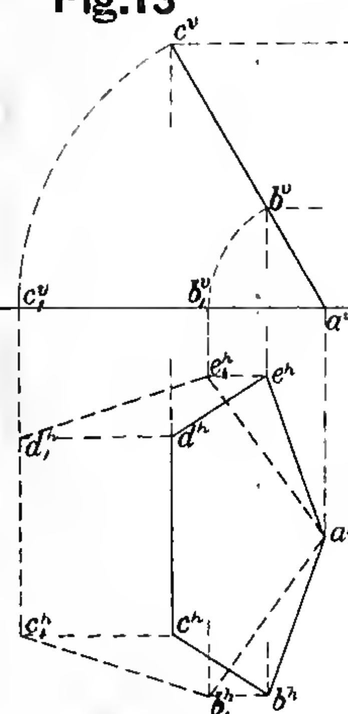


Fig. 14

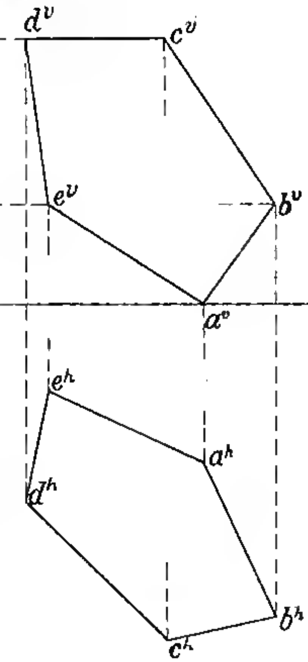


Fig. 15

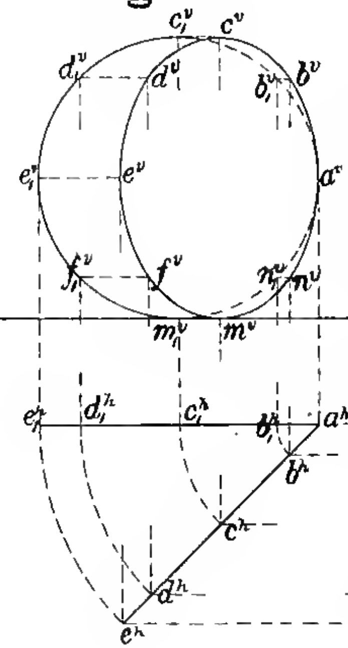


Fig. 16

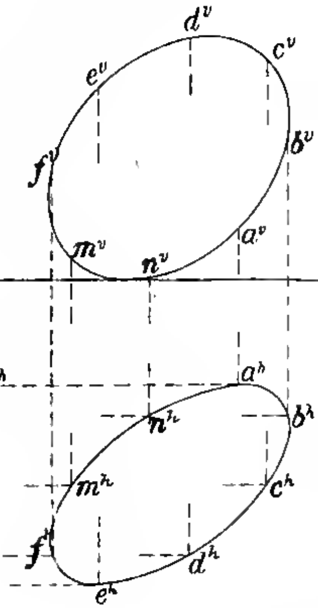


Fig. 17

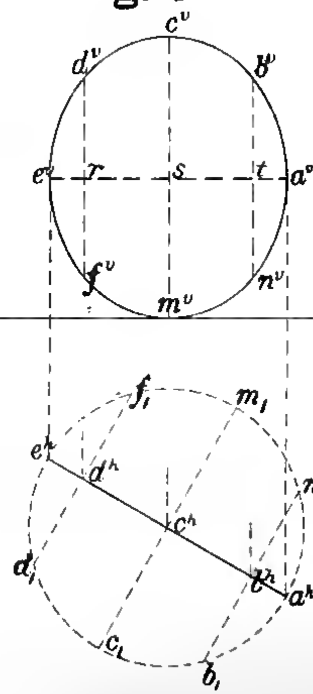


Fig. 18

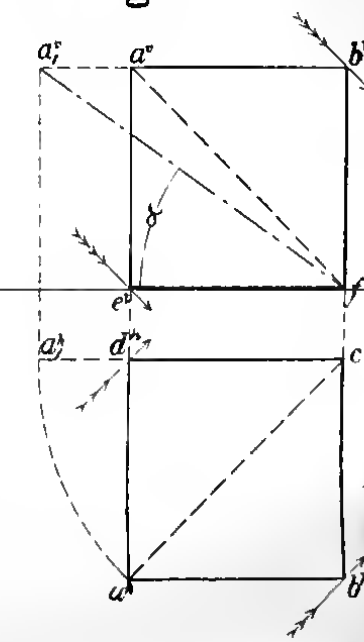


Fig. 19

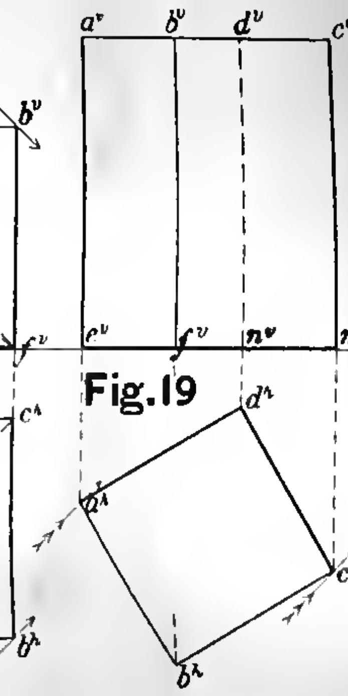


Fig. 20

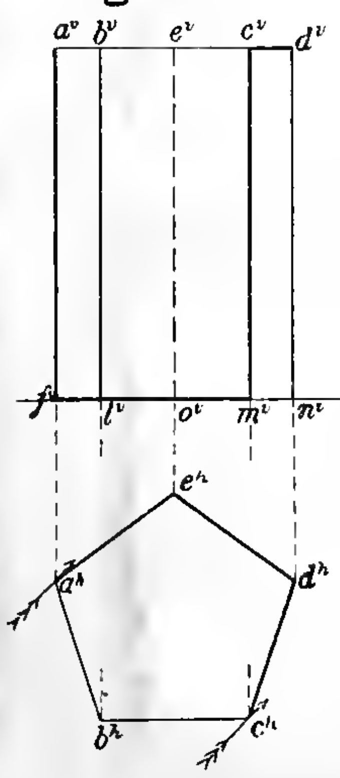


Fig. 21

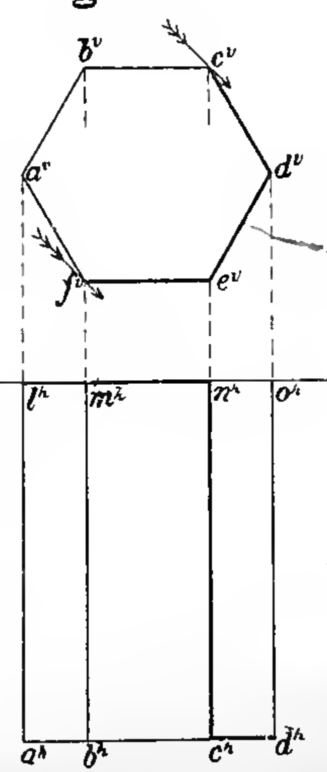


Fig. 22

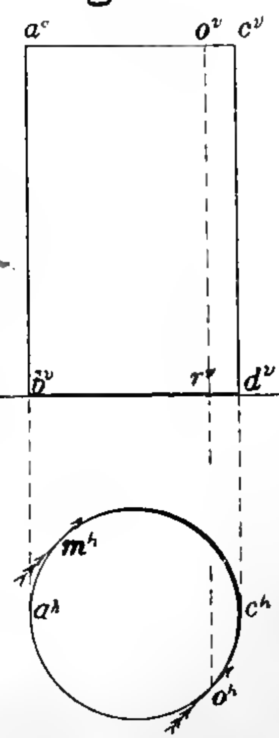


Fig. 23

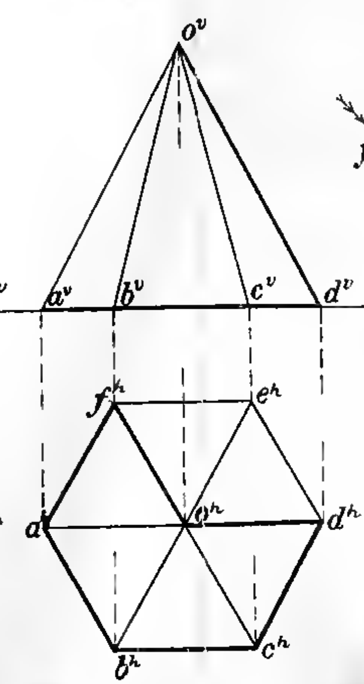


Fig. 24

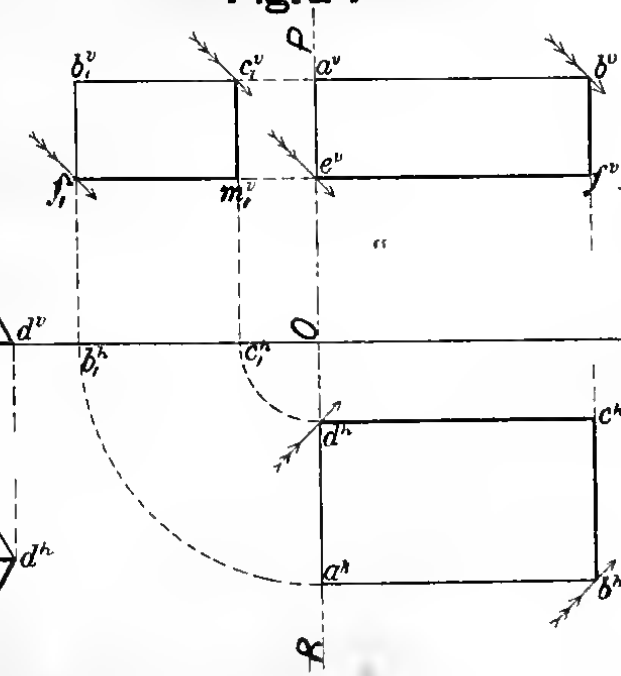
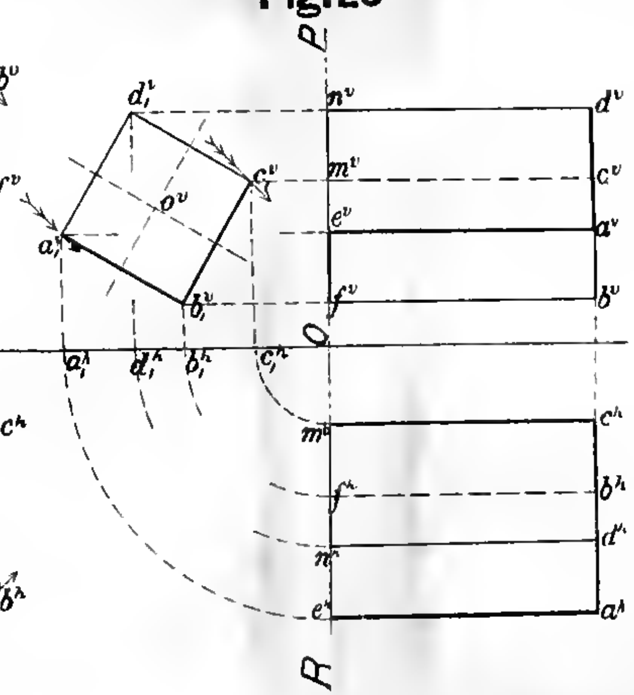


Fig. 25



If the angle the prism made with V and the angle the vertical projection made with GL had been given, the principles would have been just the same, only you would have first drawn it with its axis perpendicular to V, then revolved it about a vertical axis until it made the required angle with V; in this case the plan does not change its size; and, lastly, revolved the vertical projection last found through the proper angle and constructed the corresponding plan.

The shade lines in Figs. 30 and 31 can only be determined *positively* by casting the shadows of the doubtful surfaces. Frequently it is possible to tell which are the light and which the dark surfaces without casting the shadows by conceiving the solid in its position in space together with the ray of light, but for the average student it would be little better than a guess until he has had considerable practice in finding shadows.

Before taking up these cases where it is necessary to cast the shadow in order to determine the shade line, it will be necessary to take up so much of the subject of shadows as will enable the student to find the shadow of an ordinary object on the two coördinate planes.

79. PROB. 14. *To construct the projections of a right heptagonal pyramid when its axis makes a compound angle with the coördinate planes, the angle it makes with* H, *the angle the horizontal projection makes with* GL, *and the size of the pyramid being given.*

A careful examination of Figs. 32, 33, and 34 will be sufficient to understand this problem, since the principles are exactly the same as in the last problem.

80. PROB. 15. *To draw the projections of a prism* 1″ *square and* $1\frac{3}{4}''$ *long, resting with one of its long edges on* H, *this edge*

making an angle of 60° *with* V, *backward and to the right, its front end being* $2\frac{3}{4}''$ *in front of* V. *The lower left-hand long face making an angle of* 30° *with* H.

Also, draw projections of a regular hexagonal pyramid whose altitude is 3″, *diameter of circumscribed circle about base is* $1\frac{1}{2}''$. *The lower end of right-hand element of pyramid rests on* H $1\frac{1}{4}''$ *to the left of the point located in prism, and* $1\frac{7}{8}''$ *in front of* V; *this element also rests on top edge of prism at a point* $\frac{1}{4}''$ *from its front end. The axis of pyramid slopes downward, backward, and to the left. The two lower faces of pyramid make equal angles with* H. Fig. 62.

The projections of the prism are drawn as already described in Art. 74; the spaces $a^h d^h$, $d^h b^h$, and $b^h c^h$ are made respectively equal to $a_{,,} d_{,}$, $d_{,} b_{,,}$, and $b_{,,} c_{,,}$.

Heretofore, in drawing the projections of an object making a compound angle with the coördinate planes, we have had given the size of the object, the angle it made with V or H, and the angle its other projection made with GL, and the object has been drawn in three distinct positions. If the third position only is wanted it is not essential that the first two be wholly drawn, nor that they be made in separate figures. After the student becomes familiar with this work so that numerous lines do not confuse him, a considerable part of the construction may be omitted. In the figure all the necessary construction lines have been left in.

Here neither the angle the pyramid makes with H, nor the angle its horizontal projection makes with GL, are given, but the projections of two points f and e of one of the elements are given, which enables the projections of this element to be drawn (indefinite in length). Revolve this line around until it is parallel to V, and lay off on it in this position the true length of the element. This line will, of course, show the true size of the angle

this element makes with H, and the horizontal projection of the indefinite line before revolution shows the angle it makes with GL. All the necessary data are now obtained, and a careful study of the figure should enable the student to understand the rest of the construction.

81. It is evident that neither the height of an object above H nor its distance in front of V affects in the least the *size and shape* of its projections. Therefore, the GL is not at all essential in drawing the projections of an object unless its distances from V and H are given, which is not customary. In *working drawings* the GL is never used. It is only used in elementary projections as an aid in understanding the principles.

CHAPTER V.

SHADOWS.

82. The shadow of a body upon a surface is that portion of the surface from which light is excluded by the body.

The source of light *may* be assumed at any point, but it is customary to assume it so that the rays of light are parallel to the diagonal of a cube, as already stated in Art. 63, in which case its two projections make angles of 45° with GL. Although rays of light diverge in all possible directions from the source, yet when this source is as far removed as the sun there is no appreciable error in calling them all parallel.

83. *The shadow of a point on any surface is where a ray of light through that point pierces the surface.*

Hence, to find the shadow of a point on a surface, draw a line through the point to represent the ray of light, and find where it pierces the surface.

84. *To find the shadow of a point on* H.

In Fig. 35 let b represent the point in space and R the ray of light passing through this point. b^v and b^h will be the two projections of the point b; $b^v r$ and $b^h t$ of the ray of light.

The shadow of the point b on H will be where R pierces H. This point being in H will have its *vertical* projection in GL, and its *horizontal* projection will be the point itself (Art. 33); being in the ray of light its two projections must also be on the projections of the ray of light (Art. 42–8th). Therefore, produce, if necessary, the vertical projection of the ray of light

till it meets GL at r, and r will be the vertical projection of the point where the ray of light pierces H; at r draw a perpendicular to GL and b_s^h, where this perpendicular intersects the horizontal projection of the ray of light is its horizontal projection, and is the shadow of the point b on the plane H.

This is shown in actual projection in Fig. 37.

85. *To find the shadow of a point on* V.

In Fig. 36 let a represent the point and R the ray of light passing through it. a^v and a^h will be the projections of the point a; $a^v r$ and $a^h t$ of the ray of light.

The shadow of the point a on V will be where R pierces V. This point being on V will have its *horizontal* projection in GL, and it will also be in the horizontal projection of R, hence at their intersection t; the vertical projection of this point, and the shadow on V, will be a_s^v where a perpendicular drawn from t to GL intersects $a^v r$.

This is shown in actual projection in Fig. 38.

86. We have heretofore supposed that the coördinate planes did not extend below or behind their line of intersection, but they can just as well be considered as extending indefinitely in both directions, as shown pictorially in Figs. 35 and 36. Then, after the vertical plane has been revolved to coincide with the horizontal plane, that portion of the paper above GL represents not only that portion of V which is above H, but that part of H which is behind V; also that portion of the paper below GL represents that part of H which is in front of V and that part of V which is below H.

Referring again to Fig. 35, we have already seen that R pierces H at b_s^h; now, if we suppose R to be produced below H indefinitely, it must pierce V at some point, since it is not parallel to V. This point is found in exactly the same way as already described in Art. 85. It does not make a particle of

difference whether it pierces V above or below H. That is, every ray of light, unless parallel to V, will pierce V at some point either above or below H, and since these points are all in V their horizontal projections must be in GL. Of course, the shadow of the point b falls on H, and does not actually fall on V, but the point can be found where it would fall if H were transparent, and it is frequently convenient to do this in finding shadows of bodies in certain positions, as we shall soon see.

Fig. 37 shows this point b_s^v in actual projection. It being on that part of V which is below H after revolution appears below GL.

Referring again to Fig. 36, it is evident that R not only pierces V at a_s^v, but also pierces H at a_s^h. a_s^h being in H has its vertical projection in GL.

Fig. 38 shows this point in actual projection. It being on that part of H which is behind V, after revolution appears above GL.

87. The following rules are evident from the foregoing: —

To find the shadow of a point on H, *produce the vertical projection of the ray of light to meet* GL; *erect a perpendicular at this point of intersection, and the intersection of this perpendicular with the horizontal projection of the ray of light will be the shadow required.*

It should be carefully borne in mind that this last perpendicular may intersect the horizontal projection of the ray of light above or below GL, depending on the location of the point in space; that is, if the point be nearer H than V the intersection will be below GL, and the shadow will actually fall on H; if the point be nearer V than H the intersection will be above GL, and the shadow will be imaginary.

To find the shadow of a point on V, *produce the horizontal projection of the ray of light to meet* GL; *erect a perpendicular*

at this point of intersection, and the intersection of this perpendicular with the vertical projection of the ray of light will be the shadow required.

Here, also, the same caution as for the last rule is applicable. But in this case if the point is nearer V than H the intersection is above GL, and the shadow actually falls on V; if the point is nearer H than V the intersection will be below GL, and the shadow will be imaginary.

The following may also be noted: —

If the horizontal projection of the ray of light meets the GL *before the vertical projection, the shadow will actually fall on* V; *if the vertical projection meets the* GL *before the horizontal, the shadow actually falls on* H.

88. Fig. 39 shows how to find the shadow of a line parallel to both V and H.

Fig. 40 shows the shadow of a line perpendicular to and resting on H.

Fig. 41 shows the shadow of a line perpendicular to H but not resting on H. In this case a part of the shadow falls on each of the coördinate planes.

From these figures the following facts may be noted: —

1*st*. *The shadow of a straight line on a plane surface is a straight line.*

2*nd*. *The shadow of a line on a plane to which it is parallel is a line parallel and equal to it in length.*

3*rd*. *The shadow of a line on a plane to which it is perpendicular coincides with the projection of the ray of light on that plane, and it is longer than the line itself.*

4*th*. *The shadow of a line on a plane may be said to begin where the line pierces that plane, either or both being produced if necessary.*

5*th*. *Since two points determine a straight line it is sufficient*

to find the shadow of two points of it on a plane surface. In case the direction of the shadow, or the point where the line meets the plane surface receiving the shadow is known, it is sufficient to construct the shadow of one other point only.

6th. *When the shadow of a line falls upon two surfaces which intersect, the shadows on the two surfaces meet at a common point on their line of intersection. This is equally true whether the two surfaces intersect at right angles to each other or otherwise.*

89. PROB. 15. *To find the shadow in* V *and* H *of a line oblique to* V *and* H, *when one end is nearer* H *than* V, *and the other is nearer* V *than* H. Fig. 42.

Let $a^v b^v$ and $a^h b^h$ be the two projections of such a line. The end a being nearer H than V its shadow will fall on H (Art. 87), and will be found, as already described, at a^h_s. The end b, being nearer V than H, its shadow will fall on V, and will be found at b^v_s. Since the shadow of one end falls on H and of the other on V, it is evident that the shadow of the line will fall partly on H and partly on V, and also that a line joining these two points could only be a line *in space*, and therefore not the shadow required.

It is essential that the two points which determine the shadow of a line should be on one and the same plane; therefore, as we have the shadow of one point of the line on each coördinate plane, it is necessary to construct the shadow of another point of the line on either of the coördinate planes. Any point could be taken, but the ends being definitely projected it is more convenient to use them. We have already found the shadow of a on H to be a^h_s; the shadow of b on H is found at b^h_s, in the same way. (This shadow we know does not actually fall on H, but it serves our purpose, which is to get the direction of the shadow of the line, just as well as if it did); $a^h_s b^h_s$ is, therefore, the

Fig.26

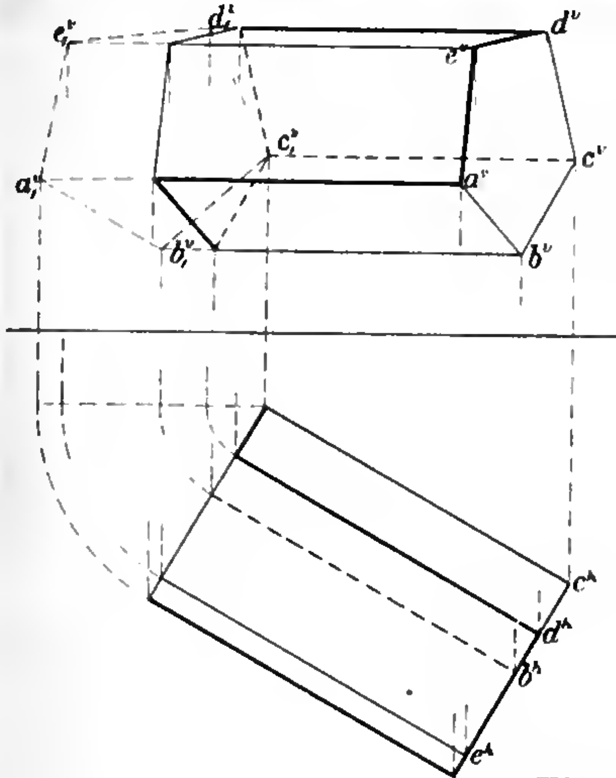


Fig.27

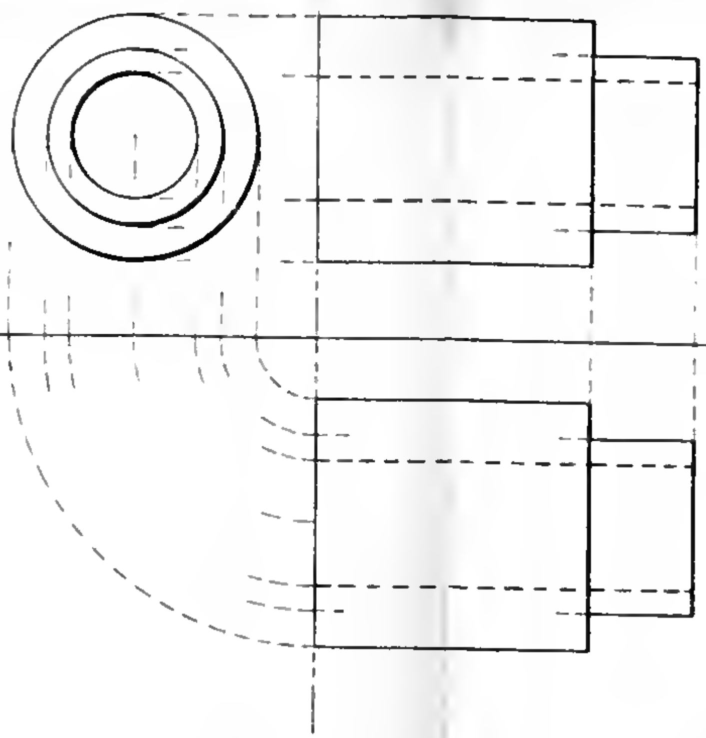


Fig.28

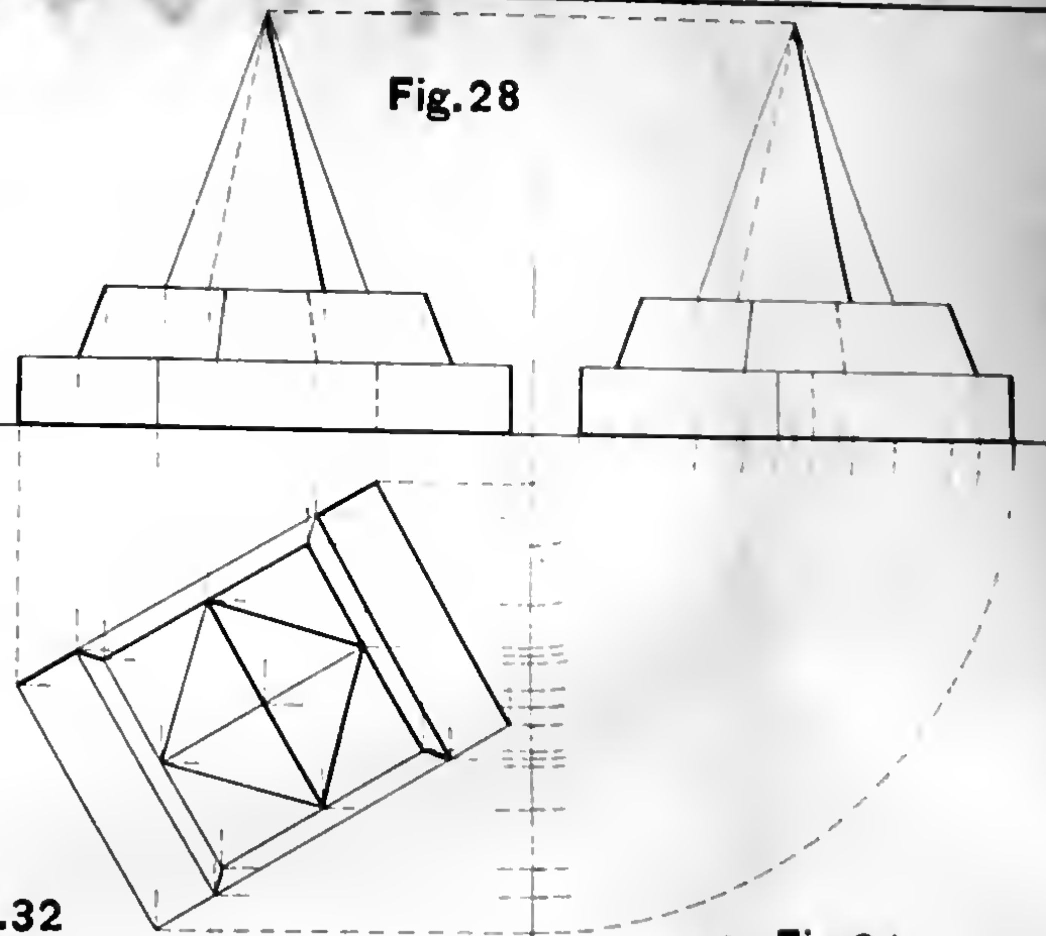


Fig. 29

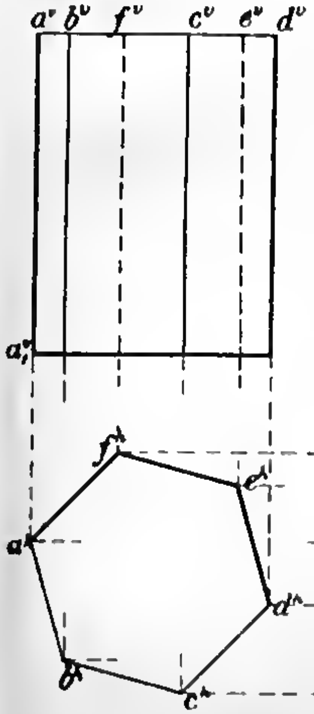


Fig.30

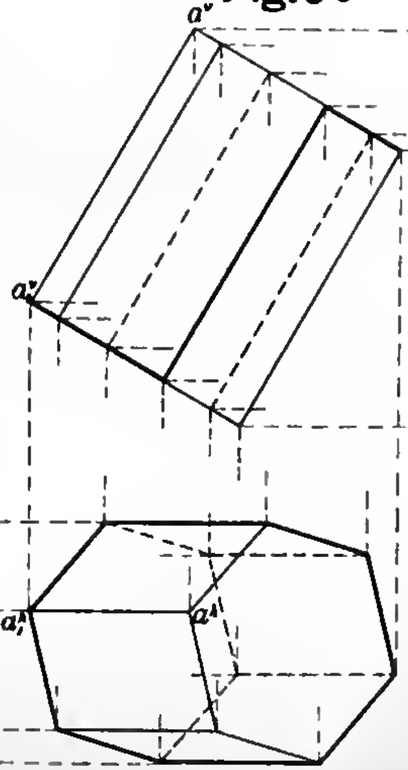


Fig.31

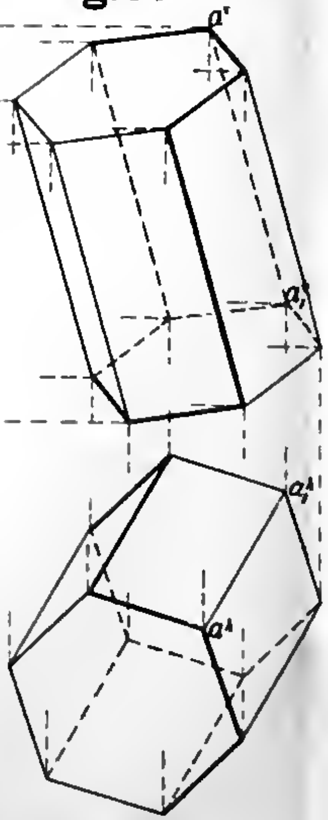


Fig.32

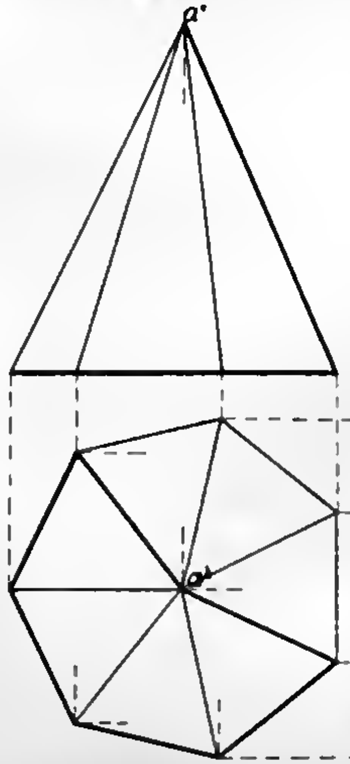


Fig. 33

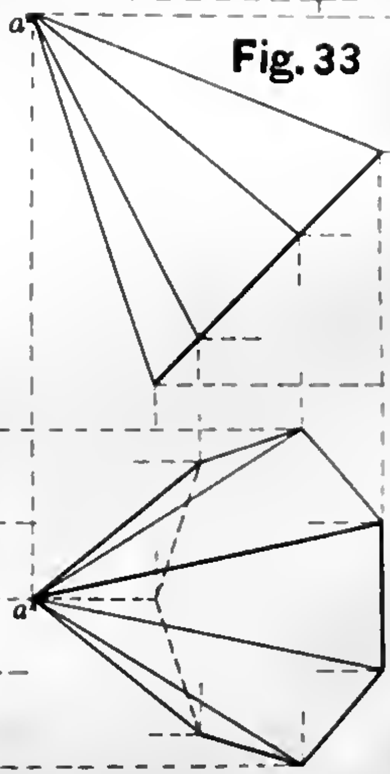
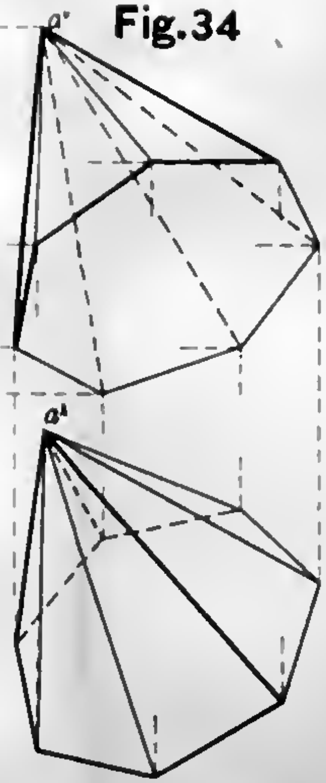


Fig.34



shadow of the *whole* line on H, but only that part of it, a_s^hc, which falls below or in front of GL is actual shadow. We have also seen that the shadow of b on V is b_s^v, the shadow of a on V is similarly found at a_s^v, and $a_s^vb_s^v$ is the shadow of the whole line on V, but only that part of it, b_s^vc, which falls above or behind GL is actual shadow. We, therefore, have as the actual shadow of the line a_s^hc on H and b_s^vc on V, the portions on the two planes intersecting in a common point, c, on GL as already noted in Art. 88–6th. Since this is the case it is evident that it is not necessary to find the shadow of the whole line on both V and H; having found the point c where either shadow crosses GL, join this point with the shadow of the end which falls on the other plane.

90. Fig. 43 shows the shadow on V of a square card whose surface is parallel to V. Since the edges of the card are parallel to V their shadows will be parallel and equal each to each, and consequently the shadow of the card will be equal and parallel to the card itself. This will be true whatever the size or shape of the card.

Therefore, *the shadow of any plane figure on a surface to which it is parallel is a figure equal and parallel to it.*

91. Fig. 44 shows the shadow on V of a square card whose surface is perpendicular to V and parallel to H. Here the two edges *ab* and *cd* are parallel to V, consequently their shadows will be equal and parallel lines. The other two edges, *ad* and *bc*, are perpendicular to V, hence their shadows make angles of 45° with GL.

92. Fig. 45 shows the shadow on both V and H of a square card whose surface is perpendicular to both V and H, that is, is in a profile plane. The edge *cd* is parallel to V, and is nearer to V than H, hence its shadow is on V parallel and equal to *cd*; the edge *ad* is nearer to V than H, and is perpendicular to

V, hence its shadow is on V, and makes an angle of 45° with GL; the edge ab is parallel to V and perpendicular to H, the upper end is nearer V than H, and the lower end is nearer H than V, hence its shadow is partly on V and partly on H; that portion which falls on V will be parallel to $a^v b^v$, and that portion which falls on H will make an angle of 45° with GL; the other edge bc is parallel to H and perpendicular to V, the front end is nearer H than V, and the back end is nearer V than H, hence its shadow will fall partly on V and partly on H; that portion which is on H will be parallel to $b^h c^h$, and that portion which is on V will make an angle of 45° with GL. The shadow of the whole card on V is $a^v_s b^v_s c^v_s d^v_s$, of which only that portion $a^v_s nmc^v_s d^v_s$, which is above GL, is visible. The shadow of the whole card on H is $a^h_s b^h_s c^h_s d^h_s$, of which only that portion $b^h_s mn$ is visible. It is, of course, not essential to find that portion $a^h_s nmc^h_s d^h_s$ of the shadow on H which is above GL.

93. Fig. 46 shows the shadow of a card lying in a profile plane, all of its edges being oblique to both V and H. Each point being found the same as all the preceding ones, no further explanation is necessary.

94. Fig. 47 shows the shadow of a circular card parallel to H. The shadow on H we know must be a circle equal in size to the card, therefore it is only necessary to find the shadow of its centre o. This is found at o^h_s. With this point as a centre and a radius equal to that of the card describe the arc of a circle mrn. Since a part of the circle is above GL it is evident that that part of the shadow actually falls on V. To get this shadow take any points on the circle, as a,b,c,d, etc., and find their shadows separately, joining these points by a curved line. The points m and n, where the circle described from o^h_s as a centre crosses GL, will, of course, be two points on the curve. This curve will be an ellipse, and *the shadow of a circle on a plane to which it is perpendicular or oblique will be an ellipse.*

95. As a solid is composed of planes, planes of lines, and lines of points, it is evident that the shadow of the most complex body is obtained by finding the shadow of one point at a time by the methods already given until the shadows of all the points on the object which cast shadows have been found, so that the student who finds himself now able to cast the shadow of any single point on a given plane has practically mastered the subject, and if such a one has any difficulty in finding the shadow of any object the trouble is that he does not understand thoroughly the subject of projections.

96. Since the shade lines of a body separate its light from its dark surfaces, the shadow of the shade lines will form the boundary of the shadow of the body. Therefore, in finding the shadow of a body the shade lines should first be marked, if it is in such a position that they can be found by means of the 45° triangle, and the shadows of these lines give the shadow of the whole object. If the object is in such a position that the shade lines cannot be found by means of the 45° triangle directly, the shadow of every point on the object, except those which it is *known* do not cast shadows, should be found separately, and then join those points which will enclose the largest area. The shade lines can then be found from the boundary of the shadow by finding what lines on the object cast these boundary lines.

97. PROB. 16. *To find the shadow of an hexagonal prism with its two ends parallel to* H. Fig. 48.

The shade lines of the prism in this position are found directly by means of the 45° triangle to be *ab*, *bc*, *cd*, *de*, *ef*, *fm*, *mn*, and *an*. It is only necessary to find the shadow of each of these lines and join them in order, and the shadow is completed. The first three and last three of these lines are

parallel to H, hence their shadows on H will be equal and parallel respectively to the edges casting them.

98. Fig. 49 shows the shadow of a square prism on V and H, resting with its base on H and its long faces oblique to V. The shade lines are found first here.

99. Fig. 50 shows the shadow of a cylinder on V and H, with its base resting on H. The shadow of any number of points on the shade line between a and d can be found. That portion of the curve between a_s^v and d_s^v will be a semi-ellipse, and the lines $a_s^v m$ and $d_s^v n$ must be tangent to this ellipse at the points a_s^v and d_s^v.

100. Prob. 17. *To find the shadow of a right cone resting with its base on* H. Fig. 51.

It is evident that, unless all the sloping part of the cone is light, there will be two elements of the cone which separate light from dark surfaces, and also that these two elements meet at the vertex and terminate at the other end in the base. But we do not know just where the light surface stops and the dark surface begins, as we do in the case of the cylinder, so we have to cast the shadow first.

The shadow of the vertex o is o_s^h; from o_s^h draw two lines $o_s^h a^h$ and $o_s^h b^h$ tangent to the base of the cone, and these will be the shadows of the two shade elements. We now see that oa and ob are the dividing lines between the light and dark surfaces, but since these lines do not coincide with the contour elements od and oc, it is evident that neither od nor oc is a shade line.

The bottom of the cone is dark and the sloping surface $o^h a^h c^h b^h$ is light, hence the edge $a^h c^h b^h$ is a shade line.

Note the difference in the shade lines on the cylinder and the cone.

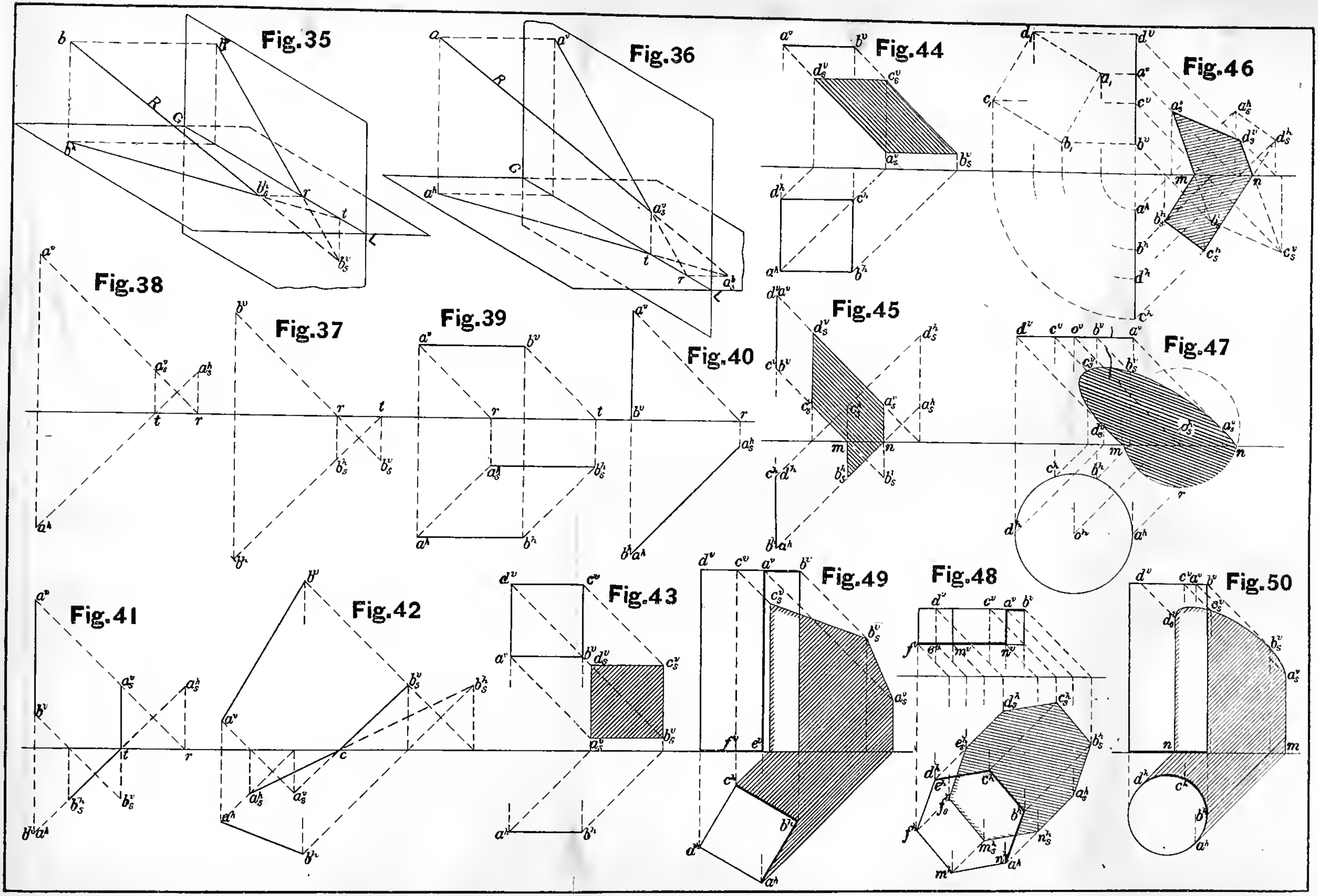


Fig. 35

Fig. 36

Fig. 37

Fig. 38

Fig. 39

Fig. 40

Fig. 41

Fig. 42

Fig. 43

Fig. 44

Fig. 45

Fig. 46

Fig. 47

Fig. 48

Fig. 49

Fig. 50

Do not shade the contour elements of a cylinder or cone.

101. We have seen that to find the shadow of a body three things must be given, the body casting the shadow, the surface receiving the shadow, and the ray of light, and that these must be given by their projections. The surface receiving the shadow has for convenience so far been taken as one of the coördinate planes, one projection of which is the plane itself and the other is the ground line, but it is more frequently necessary to find shadows of objects upon themselves and upon other objects in the immediate vicinity. We have also seen that in finding shadows on the coördinate planes we have only concerned ourselves with that projection of the surface which is a line, that is, the GL. Hence, to find the shadow of an object on surfaces other than the coördinate planes, we have only to find the line which corresponds with GL and proceed according to the rule already given (Art. 87).

To find this line, which we may call GL, observe the following rule: —

RULE FOR GL. *The* GL *to be used in finding shadows is always that projection of the surface receiving the shadow which is a line. This line may be straight, curved, or otherwise.*

Of course, this rule is applicable only when one projection of the surface is a line.

102. PROB. 18. *To find the shadow of a stick of timber on the top and front of an abutment on which it rests.* Fig. 52.

The shade lines of the stick of timber are easily found to be as follows: $cd_{,}$ $d_{,}d$, de, and ef. First find the shadow on the top surface of the abutment; the vertical projection of this surface is the line a^vb^v, which, according to the rule, is the GL to be used. The shadow of $d_{,}d$ is $d^hd^h_s$, of de is $d^h_se^h_s$, and of ef (since ef is parallel to the top of the abutment) is a line through

e^h_s parallel to $e^h f^h$. Next find the shadow on the front surface of the abutment; the horizontal projection of this surface is the line $a^h b^h$, which must therefore be the GL for this surface.

The shadow of the shade edge ef will fall on both the top and front surfaces, which intersect in the line ab, therefore the shadows on the two surfaces will meet in a common point m on this line (Art. 88–6th). m^v is, then, one point of the shadow of the edge ef on the front of the abutment. The point n^v, for the same reason, is one point of the shadow of the edge $cd_{,}$ on this surface. Since these two edges are parallel they will cast parallel shadows, hence it is only necessary to find the shadow of one more point, and the shadow is determined. The shadow of the point c is c^v_s; join this with n^v, and through m^v draw a line parallel to $c^v_s n^v$, and the shadow is completed. Of course, any other point might have been taken on the edge $cd_{,}$, or any point on the edge ef.

The irregular line $r^h s^h$ simply indicates that the abutment extends backward farther than it was necessary to show. The ragged end of the stick of timber indicates the same thing.

103. Prob. 19. *To find the shadow of a stick of timber on another stick of timber into which it is framed.* Fig. 53.

It is evident that the sloping stick will cast a shadow on the top of the horizontal stick. The lower front and the upper back edges cd and ab are the lines which cast the shadows, and mn is the line to be used as GL. The shadow of the point a is at a^h_s, apparently on the elevation of the object, but in reality it is where the shadow would be if the lower stick of timber were sufficiently wide to receive it. It is frequently necessary to imagine surfaces indefinite in extent for convenience in construction. The shadow of the edge ab begins where it leaves the surface (Art. 88–4th), therefore, join a^h and b^h and that

portion of the line which falls on the surface of the stick will be all that is necessary; through c^h draw a line parallel to $b^h a^h$, and the shadow is completed.

104. PROB. 20. *To find the shadow of one oblique stick of timber on another, both being parallel to* V *and lying against each other.* Fig. 54.

The shadow will fall on the front and top of the back stick. The ground line for finding the shadow on the front surface is *rs*, and for the top surface is *mn*. Here, as in Prob. 18, the horizontal projections of the points c^v and d^v, where the shadow leaves the front surface, will be points of the shadow on the top. A careful examination of the figure will make further explanation unnecessary.

105. PROB. 21. *To find the shadow of a straight wire lying on top of a vertical cylindrical wall on the wall and the shadow of the wall on itself.* Fig. 55.

Here the GL is a curved line, and is to be used just the same as heretofore. There being no new principles the student should be able to understand this problem from an examination of the figure.

106. PROB. 22. *To find the shadow of the head of a bolt on its shank when the length of the bolt is parallel to both* V *and* H.

In Fig. 56 the head is hexagonal and the shank is cylindrical, and in Fig. 57 the head is octagonal and the shank is hexagonal.

In cases like these it is evident that neither the plan nor elevation of the surfaces receiving the shadow is a line, but the end view, or the projection on a profile plane, *is* a line, and accord-

ing to the rule is, therefore, the GL to be used. *mno* is the GL in Fig. 56, and *mnor* in Fig. 57. The only other new point to be noticed is that the two elevations of the ray of light each make an angle of 45° with a horizontal line, and slope in the same direction.

107. Prob. 23. *To find the shadow of a chimney or tower of a house on the roof.* Fig. 58.

The end view of the roof *mn* is the GL for this problem. The shadow is found the same as in Figs. 56 and 57.

108. Prob. 24. *To find the shadow of a stick of timber on the top and sloping faces of an oblique abutment.* Fig. 59.

Since neither plan, elevation, nor end view of these sloping surfaces is a line our rule is no longer directly applicable, and we may make use of an indirect method (the shadow can be found *directly* by descriptive geometry methods).

First find the shadow on top of the abutment. The points c^h and e^h, where it leaves the top, will be two points of the shadow on the left, front, sloping face (Art. 88–6th), c^v and e^v will be their vertical projections; in the same way d^h and n^h will be two points of the shadow on the right, front, sloping face, and d^v and n^v will be their vertical projections. The shadow of the lower front edge (the upper back edge could just as well have been taken) on the *ground* is the indefinite line f^ho^h. The ground and the sloping face of the abutment intersect, hence the point f^h will be a point of the shadow on the left front face; f^v will be its vertical projection; for the same reason o^h is a point of the shadow on the right, front face, and o^v is its vertical projection. Join e^hf^h, and through c^h draw a line parallel to it; also, n^ho^h and a line through d^h parallel to it. This completes the shadow on the plan. The shadow on the elevation is found by projection from the plan.

If the abutment had been so high that the shadow on the ground would have been difficult to obtain, an imaginary horizontal plane could have been taken at any convenient place, and the shadow found on that, noting where it comes out from the abutment. The dash and two dots line in plan and elevation represents the two projections of the intersection of such an imaginary plane with the abutment. m^ht^h is the shadow of the lower front edge of the stick on this imaginary plane, which gives the points t^h and m^h as points of the shadow. The shadow is completed as before.

109. PROB. 25. *To find the shadow of any oblique line on any oblique plane.* Fig. 60.

First find the shadow on any horizontal auxiliary plane. m^ve^v will be the vertical projection of one such plane, and will be the GL for this plane. The shadow of any point as a on this plane will be a^h_s; the shadow of any other point could be found in the same way, thus getting the shadow of the whole line, but the shadow of a line begins on a plane where the line pierces the plane, and the line pierces this auxiliary plane at the point b (b^v being where m^ve^v intersects the vertical projection of the line, and b^h being perpendicularly below it on the horizontal projection of the line), therefore, joining a^h_s and b^h we have the shadow on the auxiliary plane. m^ve^v is the vertical projection of the line of intersection of the auxiliary plane with the card *mnop*; its horizontal projection is, therefore, m^he^h. The lines $b^ha^h_s$ and m^he^h lie in the auxiliary plane, the line m^he^h also lies in the plane of the card *mnop*, therefore, the point r^h, where these two lines intersect, is a point on the card, and must be one point of the horizontal projection of the shadow required; r^v on m^ve^v is its vertical projection. Assume any other auxiliary plane, as d^vo^v, and another point, s^vs^h of the shadow, will be

found in the same way. Draw an indefinite line through these points r and s in both projections, and the shadow is finished.

Vertical auxiliary planes could have been taken instead of horizontal with the same result.

110. PROB. 26. *To cast the shadow of an abacus on a conical column.* Fig. 61.

Since neither projection of the surface receiving the shadow is a line, this problem must be done by the indirect method as used in the two preceding problems.

Find the shadow on any horizontal plane, as $a^v c^v$. Its vertical projection is $a^v c^v$, and is the GL for this shadow. This plane cuts the column in a horizontal circle, of which $a^v c^v$ is the vertical and $a^h b^h c^h$ is the horizontal projection. The shadow of the bottom edge of the abacus on this plane is the circle $d^h a^h e^h$, drawn with o^h_s, the shadow of o the centre of the abacus, as a centre and radius equal to that of the abacus. This circle cuts the circle $a^h b^h c^h$ at the point a^h, which is one point of the horizontal projection of the shadow required. a^v, on the vertical projection of this auxiliary horizontal plane, is one point of the vertical projection of the required shadow. Any number of other points can be found in the same way.

111. PROB. 27. *To cast the shadow of the prism given as in* Fig. 62 *on* H *and* V, *also of the pyramid on the prism and on* H.

The shadows of the prism on H and V and of the pyramid on H require no explanation.

If the shadow of a line falls partly on two plane surfaces, A and B (no figure), which do not intersect (at that portion under discussion), A being between the line and B, that part which falls on A does not fall on B, and the shadow of the line

on B may be said to begin at the shadow of the point where the shadow of the line leaves A, on B. Now, if B is horizontal or vertical, and A is oblique to both V and H, the shadows of the line and the plane A on B can be readily found; and then get the shadow of the line on A by the reverse of the above process, that is, note where the shadow of the line and plane A on B intersect; find what point on A cast this intersecting point, and that will be one point of the shadow of the line on A.

Now, referring to Fig. 62, we see that the shadow of the pyramid and prism on H intersect at the four points r^h_s, t^h_s, x^h_s, and z^h_s; the points on the prism (this being between the pyramid and H) which cast these shadows are r^h, t^h, x^h, and z^h respectively (found by drawing a 45° line from the shadow to the edge casting it), and they are, therefore, points of the shadow of the pyramid on the prism.

The shadow on the upper edge of the prism may be found by this same principle, that is, the shadow of this upper edge on H is $b^h_s y^h_s$; this intersects the shadow of the pyramid on H at s^h_s and y^h_s; s^h and y^h are the points which cast these shadows, hence they are points of the shadow required. Join $r^h s^h$, $s^h t^h$, $x^h y^h$, and $y^h z^h$, and the horizontal projection of the shadow is completed. Its vertical projection is found by projecting each one of these points on to the vertical projection of the prism. The points s^h and y^h could also have been found by casting the shadow of the pyramid on an auxiliary horizontal plane through the top edge of the prism; $o^h_{,s} s^h$ and $o^h_{,s} y^h$ are the shadows of the two shade elements of the pyramid on such a plane; they intersect the upper edge of the prism at the points s^h and y^h, which are the points required. A vertical auxiliary plane through this edge or the other edges of the prism might have been used with the same result.

CHAPTER VI.

ISOMETRICAL DRAWING.

112. In all the previous constructions two projections have been used to represent a body in space. In isometrical projections only one view is used, the body being placed in such a position that its principal lines or edges (length, breadth, and thickness) are parallel to three rectangular axes, which are so placed that equal lengths on them are projected on the plane equal to each other. Thus we have the three dimensions of a body shown on one plane in such a way that each can be measured, thereby combining the exactness of ordinary projections and the intelligibleness of pictorial figures. It is used chiefly to represent small objects in which the principal lines are at right angles to each other. In large objects the drawing would look distorted.

113. If we take a cube situated as in Fig. 63, and tip it to the left, about its lower left corner e until the diagonal cg is horizontal, Fig. 64, and then turn it through an angle of 90°, still keeping cg horizontal, we obtain Fig. 65. The vertical projection in this figure is what is called an isometrical projection.

The edges of a cube are all of equal length, and it will be seen that they appear equal in the figure, consequently the visible faces must appear equal. It will also be seen that the figure can be inscribed in a circle, and that the outline of the isometrical projection is a regular hexagon, hence, that those lines

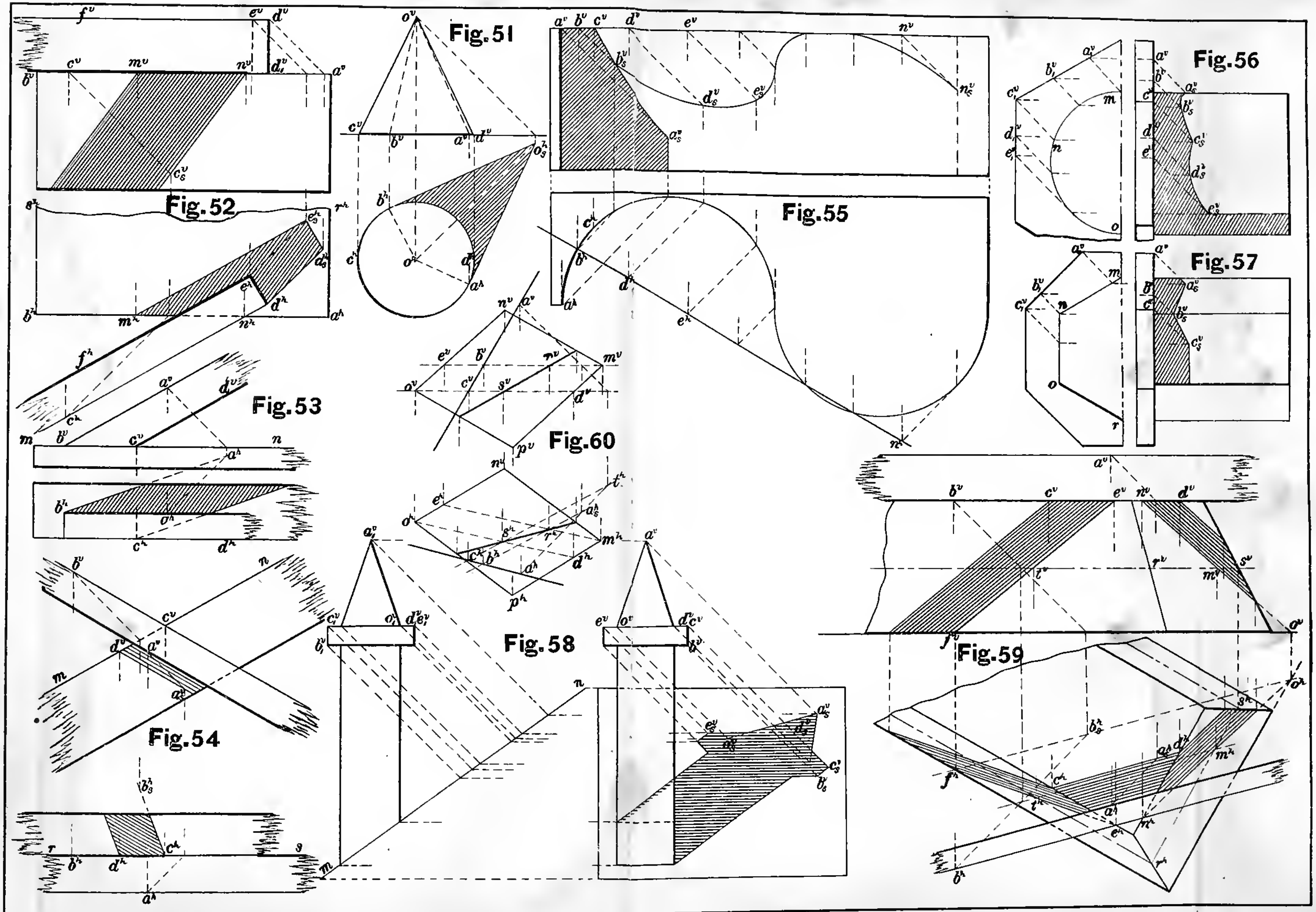


Fig. 51
Fig. 52
Fig. 53
Fig. 54
Fig. 55
Fig. 56
Fig. 57
Fig. 58
Fig. 59
Fig. 60

which represent length and breadth make angles of 30° with a horizontal, and those which represent thickness are vertical.

114. The edges of the cube being inclined to the plane on which they are represented, appear shorter than they actually are on the object, but since they are all *equally* foreshortened, and since a drawing may be made to any scale, it is customary to ignore this foreshortening, and make all the isometrical lines of the object equal to their true lengths. This will give what is called the *isometrical drawing* of the object, which will be somewhat larger than the *isometrical projection.*

Fig. 65 represents the isometrical projection of the cube shown in Fig. 63, and Fig. 67 is the isometrical drawing of the same cube.

115. DEFINITIONS. c^v, Fig. 65, is the *isometric centre.* c^vb^v, c^vd^v, and c^ve^v are the *isometric axes.* Lines parallel to either of the isometric axes are *isometric lines*, while any line not parallel to one of these axes is a *non-isometric line.* Planes parallel to the faces of the cube are *isometric planes*, and those which are not parallel to one of these faces are *non-isometric planes.*

116. DIRECTION OF LIGHT. In isometrical drawing the light is supposed to be parallel to a diagonal of the cube, as in ordinary projections, only here it is parallel to the plane of projection, and it is represented by a line making an angle of 30° with a horizontal line. Any line parallel to *df*, Fig. 67, may represent a ray of light.

117. SHADE LINES. These are the same as in ordinary projections, that is, they separate light from dark surfaces. In all rectangular objects the top, left front, and left back sur-

faces are light, while the bottom, right front, and right back surfaces are dark. In Fig. 67 the edges *ab*, *bc*, *ce*, and *el* are the visible shade lines. And all rectangular objects have their shade lines in relative positions to those of the cube. As in ordinary projections, in putting shade lines on a group of objects touching each other, the group is shaded as if it were one solid; also, in outline drawing the shadows are disregarded in putting in shade lines.

118. Prob. 28. *To make the isometrical drawing of a cube.* Fig. 67.

With the centre *c* and radius equal to the edge of the cube draw a circle, and in it inscribe a regular hexagon, and draw the alternate radii *cb*, *cd*, and *ce*, and the drawing is completed.

Another method, which is applicable to any rectangular object, is to draw from any point as *e* lines *ef* and *el*, each making an angle of 30° with a horizontal, and the vertical line *ce*; on these lay off *ef*, *el*, and *ec* equal to the true length of the edge of the cube; from the points *f* and *l* draw indefinite vertical lines; from *c* draw the lines *cb* and *cd* parallel to *ef* and *el*, intersecting the verticals through *f* and *l* in the points *b* and *d*; from the points *b* and *d* draw lines parallel to *cd* and *cb*, meeting in *a*. This completes the drawing.

119. Prob. 29. *To make the isometrical drawing of a rectangular block, with another rectangular block resting on its top face, and a recess in its right front face.* Fig. 66.

Construct the isometrical drawing of the large block by the second method of Art. 118.

To draw the small block it is first necessary to locate one of its lower corners in the top face of the large one. This must be done by means of two coördinates referred to two isometric

lines as axes. The point e, in the top face of the large block, is $\frac{1}{8}''$ from the side and $\frac{3}{16}''$ from the end, therefore make af equal to $\frac{1}{8}''$ and ag equal to $\frac{3}{16}''$, then from f and g draw the isometric lines ef and ge, intersecting in e, the point required. The rest of the small block is drawn in the same way as the large one.

To make the recess in the front side the point t is located in the same way as the point e, and tx, equal to the depth of the recess, is laid off as shown. The rest of the construction is evident.

120. Fig. 68 shows the isometrical drawing of a rectangular box, without cover, 15″ long, 6″ wide, and 4″ high, outside measurements, the boards being $\frac{3}{4}''$ thick. The scale being $1\frac{1}{2}'' = 1'$. The ends are nailed on to the sides, and the bottom is nailed to the sides and ends. The visible joints are shown. The dotted lines show the inner edges of the box which are not visible.

121. Fig. 69 shows the isometrical drawing of a four-armed cross. A careful examination of the figure will enable the student to understand its construction, there being only isometric lines involved.

122. Prob. 30. *To make the isometrical drawing of the pentagonal prism shown in Fig.* 20. Fig. 70.

The edges of the base, not being at right angles to each other, are non-isometric lines, hence the base should first be inscribed in a rectangle. Let one side of the rectangle contain the edge bc, and the other sides respectively contain the corners a, d, and e. Make the isometrical drawing of the rectangle and locate each corner of the base, a, b, c, d, and e, by laying off on the sides of this rectangle the distance of each point respect-

ively from the nearest corner of the circumscribed rectangle. At these points draw vertical lines, and lay off on each of them the true height of the prism and join the tops, completing the drawing of the prism. It will be noticed that the non-isometric lines on the drawing are not equal to their true length on the object.

123. Prob. 31. *To make the isometrical drawing of an oblique timber framed into a horizontal one, as given in Fig.* 53. Fig. 71.

The horizontal timber is drawn as usual. To draw the oblique one, these edges being non-isometric, two points have to be located by means of coördinates. The point *a* is located by making *ad* equal to the distance the lower end of the oblique stick is from the end of the horizontal one. Any other point *b* is found by making *cd* equal to the horizontal distance of the point *b* from *d*, and *bc* equal to its vertical distance from *d*. Join *ab*, which gives the isometrical drawing of one edge of the oblique timber. The other edges are, of course, parallel to this, being drawn through the points *e* and *f*, which are located the same as point *e* in Prob. 30.

124. Prob. 32. *To make the isometrical drawing of the skeleton frame of a box made in the form of the frustum of a square pyramid.* Fig. 72.

Let a square prism be circumscribed about the frustum. The isometric of this prism is readily drawn, and is shown by the dotted lines. The bottom edge of the frustum coincides with the bottom of the prism. The points *a*, *b*, *c*, and *d* are in the upper face of the prism, and are found as the point *e* is in Prob. 30.

Join *ab*, *bc*, *cd*, *da*, *af*, *bg*, and *de*, and the main frustum is

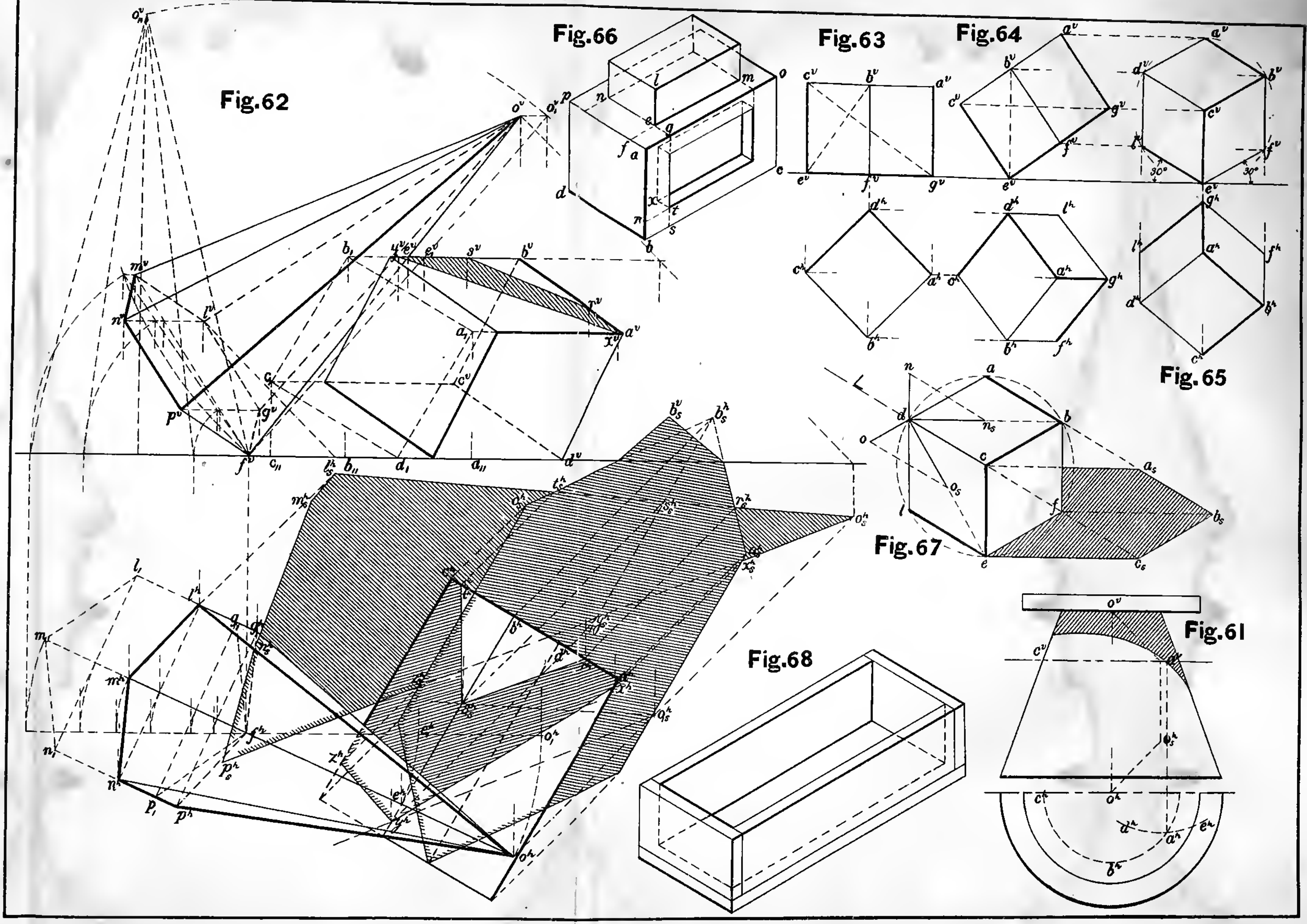


Fig. 61

Fig. 62

Fig. 63

Fig. 64

Fig. 65

Fig. 66

Fig. 67

Fig. 68

completed. The other lines which change the frustum from a solid to a skeleton need no explanation.

125. Prob. 33. *To make an exact isometrical drawing of a circular card, and also of any scroll or letter on its surface.* Fig. 74.

Let the circle and letter G be given as in Fig. 73. First, circumscribe a square about the circle. Make the isometric drawing of the square. The centres of the sides of the square, *d*, *e*, *f*, and *g*, give four points of the isometric of the circle. Each point on the circumference of the circle, as *a*, *b*, *c*, etc., has two coördinates, by means of which the isometrical drawing of the points may be easily obtained. There will be four points on the circle whose coördinates will be the same as those for *c*, and eight points whose coördinates will be the same as those for *a* or *b*, or any point between *d* and *e*. The more points that are taken the more accurate will be the ellipse which forms the isometrical drawing of a circle.

The letter is drawn in the same way by taking the two coördinates of any point on it.

126. Prob. 34. *To make an approximate construction of the isometrical drawing of a circle.* Fig. 75.

Make the isometrical drawing of the circumscribed square as before; *d*, *e*, *f*, and *g* will be four points. Draw the lines *ag*, *af*, *bd*, and *be*, intersecting in the points *c* and *o*; *c* will be the centre of the arc between *d* and *g*; *o* of that between *e* and *f*; *a* of that between *g* and *f*; and *b* of that between *d* and *e*.

127. Fig. 76 shows the approximate construction of the isometrical drawing of circles in each of the three visible faces of a cube. No explanation is necessary.

The student should study this and Fig. 75, so as to be able to make the isometrical drawing of a quarter of a circle in either isometric plane without making the whole circle.

128. Fig. 78 shows the isometrical drawing of a bolt, an hexagonal nut, and a circular washer as shown in Fig. 77.

129. Fig. 79 shows the isometrical drawing of the hollow cylinder given as in Fig. 27.

130. PROB. 35. *To divide the isometrical drawing of a circle into equal parts.* Fig. 74.

At the middle point *f* of one of the sides *nl* of the isometrical drawing of the circumscribed square draw *fo* perpendicular to *nl*, and make *fo* equal to the radius of the circle; draw *ol* and *on*; with *o* as a centre and radius *fo* describe the arc *rfs*; divide this arc into any number of parts; draw through *o* and these points of division lines to meet *nl*; join these meeting points with *t*, the centre of the ellipse, and where these lines intersect the ellipse will be the points of division of the isometrical drawing of one quarter of the circle.

The other quadrants can be divided in the same way.

ANOTHER METHOD. On the long diameter of the ellipse draw the semicircle *ckp*; divide this into any number of equal parts; through these points of division draw lines perpendicular to *cp*; where these lines intersect the ellipse will be the points of division sought.

131. In isometrical drawing the shadow of a point on a plane is where the ray of light through the point intersects its projection on that plane.

To find the shadow of the line dn on the top of the cube. Fig. 67.

A ray of light through the point *n* is nn_s, its projection on

the top of the cube is dn_s; these two lines intersect at the point n_s, which is the shadow of the point n on the top of the cube. Join this point with d, and dn_s is the shadow of the vertical line dn required.

To find the shadow of the line do on the left, front face of the cube.

A ray of light through the point o is oo_s, its projection on the face of the cube is do_s; these lines intersect at the point o_s, which is the shadow of the point on the left, front face of the cube. Join this with d, and do_s is the shadow of the line required.

132. PROB. 36. *To find the shadow of a cube on the plane of its base.* Fig. 67.

The shadow of the edge ec is ec_s; of the back edge ac is ca_s; of the point b is b_s; therefore, the shadows of ab and bc are a_sb_s and b_sc_s respectively, which completes the shadow required.

133. In Figs. 71 and 78 the shadows cast by the objects on each other and on the ground are shown.

134. To find the shadow of any point on any horizontal isometric plane proceed as follows: Draw though the point a vertical line, and make it equal in length (downward from the point) to the height of the point above the plane receiving the shadow; through the upper end of this line draw a line at 30° to a horizontal in the direction of the ray of light; through the lower end draw a horizontal line; where these two last lines intersect will be the shadow of the point.

To find the shadow of a point on any vertical isometric plane: Draw through the point a line at 30° to a horizontal, backward and to the right, and make it equal in length (backward from the point) to the perpendicular distance of the point

from the plane; through the front end of this line draw a line parallel to the ray of light; through the back end draw a line at 60° to a horizontal, forward and to the right; where these two last lines intersect will be the shadow of the point.

OBLIQUE PROJECTIONS.

135. Oblique projections differ from isometrical projections only in the position of the principal faces of the object. Instead of being placed so that its principal faces make equal angles with the plane on which it is represented, one of them is placed parallel to this plane, while the edges which represent the remaining dimension of the object may be drawn at *any* angle with a horizontal, for convenience usually at 30° or 45°. With this difference, all the statements and principles of isometrical drawing are equally applicable to oblique projections.

136. Fig. 80 shows the oblique projection of a cube, the shadow of lines on its top and front faces, the shadow of the cube on the plane of its base, and the manner of drawing the oblique projection of circles in the three faces of the cube. The ellipse in the top face is drawn by the arcs of circles, and that in the right face by points located exactly by coördinates.

It will be noticed that the ray of light is parallel to the diagonal *df* of the cube, and that its projection on a principal surface is parallel to the diagonal *db* or *de*.

137. Fig. 81 shows the oblique projection of the hollow cylinder given as in Fig. 27, of which Fig. 79 is the isometrical drawing. Figs. 1, 35, and 36 are oblique projections of models representing the principles of projections and shadows.

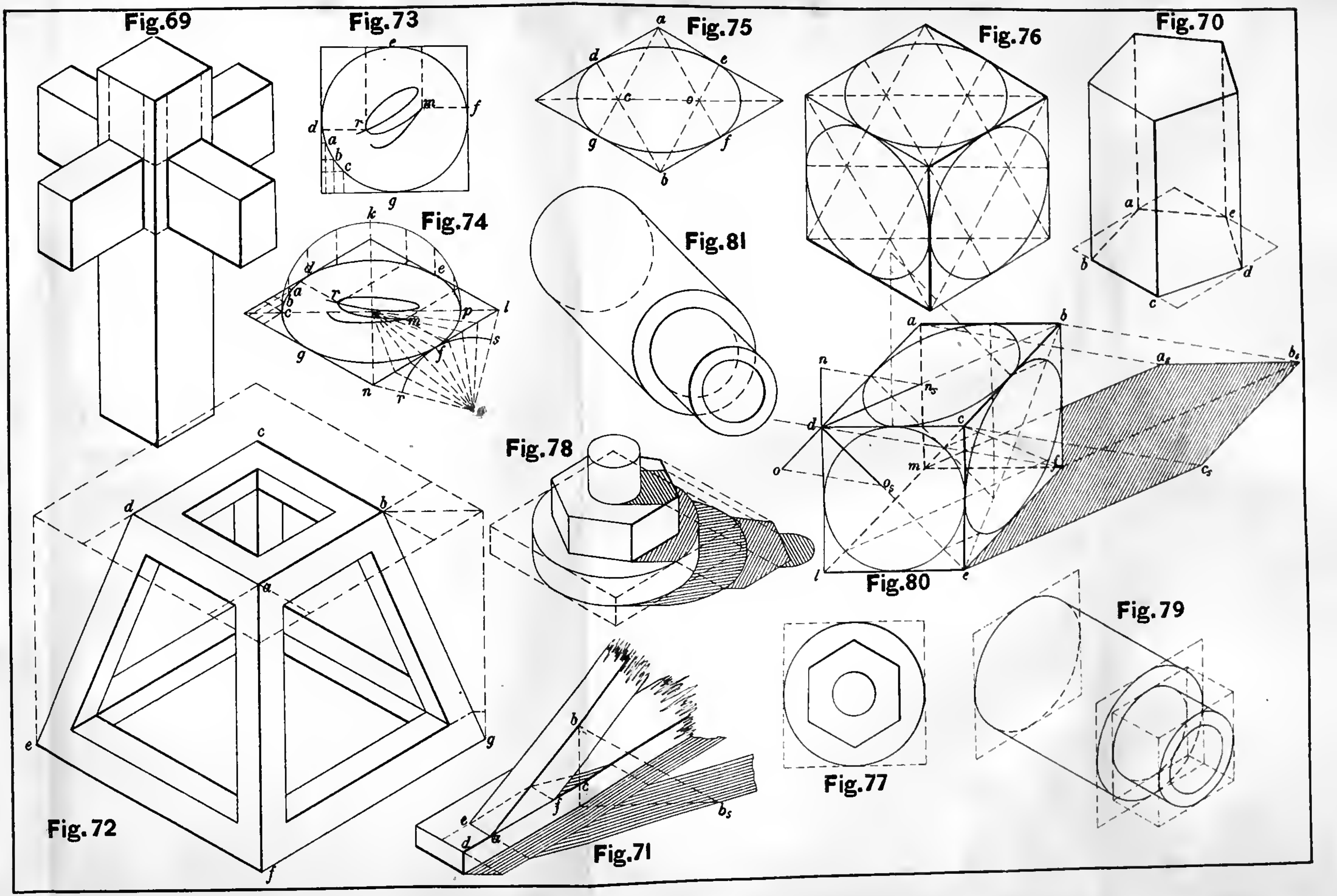


Fig.69
Fig.73
Fig.75
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Fig.70
Fig.74
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Fig.78
Fig.80
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Fig.71
Fig.77
Fig.79

CHAPTER VII.

WORKING DRAWINGS.

138. A working drawing is one which shows all the dimensions of an object in such a way that the object could be reproduced or constructed from the drawing. Two views at least, plan or horizontal projection, and elevation or vertical projection, are necessary; but more frequently a third view, usually an end view, is also necessary. Besides these views, one or more sections depending upon the object are sometimes necessary to completely determine all of its dimensions.

139. If an object is cut through, in any direction, by an imaginary plane, the projection of one part of the object on a plane parallel to the cutting plane, the person supposed to be facing this cutting plane, is called a section. Sections are taken to show the form and dimensions of the interior of a hollow object, and also of some parts of solid objects which are not completely determined by its plan and elevations. These imaginary cutting planes are *usually* taken either vertical or horizontal, but it is sometimes necessary to take them in other positions, but perpendicular to a vertical or horizontal plane. All that part of the object which is cut by these imaginary planes is cross-hatched, that is, is covered by parallel lines quite near together. The direction of these lines should be either 30°, 45°, or 60°, and from $\frac{1}{32}''$ to $\frac{1}{8}''$ apart, depending on the size of the surface to be cross-hatched, the smaller the nearer together. They may be drawn in either direction.

140. Fig. 82 represents the elevation, Fig. 83 the plan, and Fig. 84 a vertical section through the centre on the line AB, of a stuffing-box gland. The side elevation was not necessary in this case, as all the dimensions of the solid are shown without it. These figures are made one-half size.

141. It is not sufficient simply to draw the projections of the object the correct size, but the dimensions of the solid should all be clearly placed on the drawing, so that the workman, or whoever has occasion to read the drawing, is not obliged to use the scale, thereby removing a great liability to error. These dimensions should be put in neatly, and should follow a certain system. The method used in Figs. 82, 83, and 84 should be carefully observed, besides which the following general directions for putting in dimensions and representing special features should be followed:

In placing dimensions upon the drawing a line should be drawn from one point to another, between which the dimension is to be given, and the actual dimension, or distance apart of the points, is placed in the line, a space having been left for it near the centre. These lines should be fine and composed of dashes about $\frac{1}{2}''$ long with about $\frac{1}{8}''$ spaces between them. When the dimension is small of course the dashes must be made shorter. Arrow heads are placed at the ends of these lines, the point of the arrow exactly touching the points or lines between which the dimension is given, the arrow heads pointing away from each other. When the dimension is very small the arrow heads may be placed on the outside of the lines instead of between them, and in that case should point toward each other.

The arrow heads should be drawn free-hand and not made with the drawing pen. The figures for dimensions should also be made free-hand, and should always be placed at right angles

to the dimension line, and should read from the bottom or right-hand side of the drawing. They should be put down in inches and thirty-seconds, sixteenths, eights, quarters, and halves, as the case may be, thus, $1\frac{5}{16}''$, not $\frac{21}{16}''$. The fractions should be reduced to their lowest terms, thus, $\frac{5}{8}''$, not $\frac{10}{16}''$. The dividing line in the fraction should always be made *horizontal*, as they are less likely to be misunderstood. The inch marks should be placed after the fraction.

On rough castings measure to the nearest sixteenth, on ordinary finished surfaces take the nearest thirty-second, and on fine finish and fits be as accurate as possible.

All dimensions up to two feet should be put down in inches, thus, 15″, $22\frac{3}{4}''$, and all above that in feet and inches, thus, 3′–2″, or 3 ft. 2″. Students should be very careful to get all of the *important* dimensions on, and also an "overall" dimension, so that the workman will not be required to add a number of dimensions together. Important dimensions are those which are necessary for the workman to construct the piece. The dimensions should not interfere with each other, and care should be taken not to have them cross each other in a circle.

As a general thing do not repeat any dimensions; that is, if a dimension is given on one view do not repeat it on another.

In order that the drawing may be left as distinct as possible it is frequently advisable to put the dimensions outside the figure, or better, where two or more views are given, put them between the different views, as shown in Figs. 82, 83, and 84; the widths being placed between the plan and elevation, and the heights between the elevation and section, or between the two elevations where an end elevation is given. To do this "extension" lines must be used. They should be composed of fine dash lines, the dashes being about $\frac{1}{8}''$ long, so as to be distinguished easily from the dotted lines representing the invisi-

ble parts of an object. Where the dimensions do not interfere with the drawing, as is the case in Fig. 84, it is better to put them on the figure between the lines themselves, or as near as possible.

Give the diameter of a circle instead of the radius. When only an arc is shown give the radius, and draw a very small circle about its centre, and let this circle take the place of an arrow head. The dimension line should be drawn from the edge of this circle and not from its centre.

In locating holes or bolts the dimensions should be given from the outside of the piece to the centre of the hole or bolt, and their distance apart is shown by giving the distance between centres. See Fig. 85. Holes are very often located from the centre line of a piece, so that it is unnecessary to give the dimension from the outside of the piece.

The "centre" line should be composed of long and short dashes, the long dashes being about $\frac{1}{2}''$ long and the short ones about $\frac{1}{8}''$ long.

If the holes are arranged in a circle, as in Fig. 86, give the diameter of the circle passing through the centre of the holes.

In drawing a bolt or screw represent the threads as shown in Fig. 87 ; it is not necessary that the spaces should correspond with the true pitch of the threads. In order to obtain the correct slant of the threads, a line drawn at right angles to the axis of the bolt should pass through the point of a thread on one side, and the centre of a space on the other.

Always give that view of a square, hexagonal, or octagonal bolt-head, or nut which shows the distance between its parallel sides.

In placing the dimensions upon a bolt or screw, always give the length of the unthreaded part in addition to the length of the bolt. The length of the bolt should be given from the under side of the head to the extreme end. See Fig. 87.

Fig.82

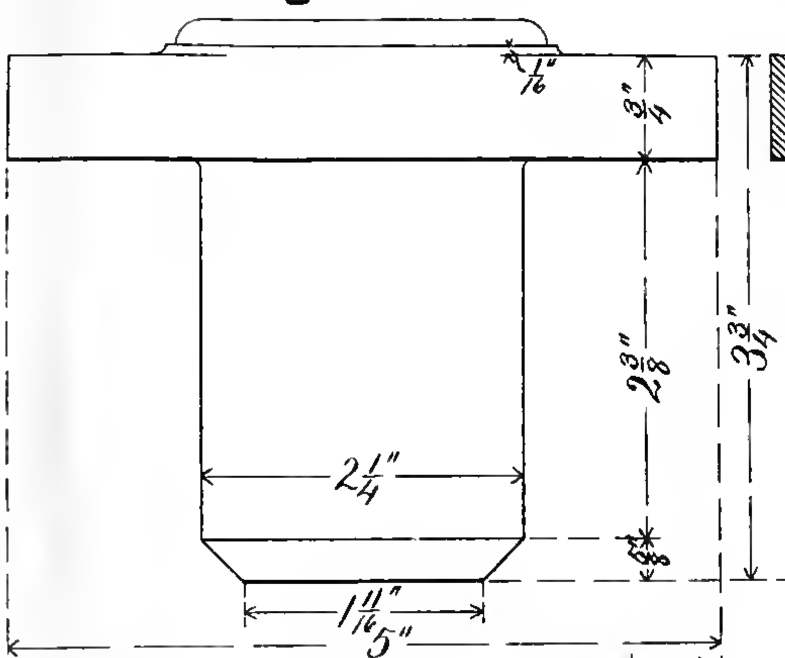


Fig.84

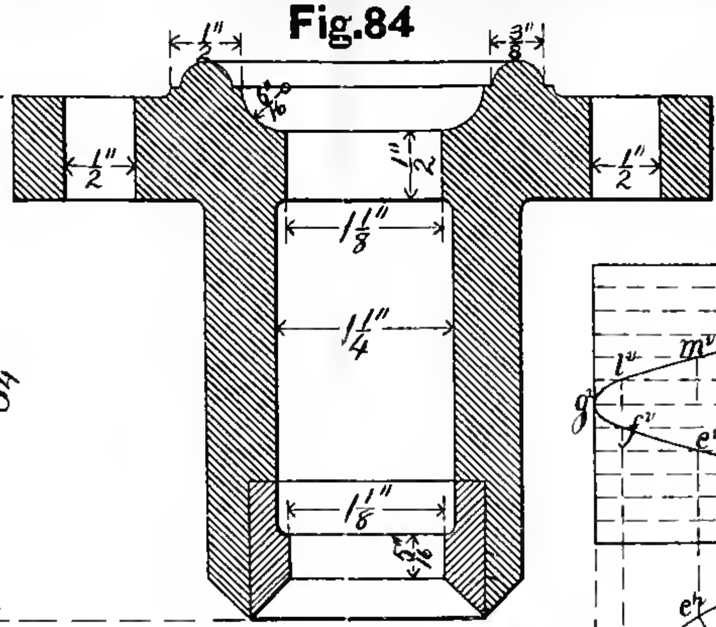


Fig.92

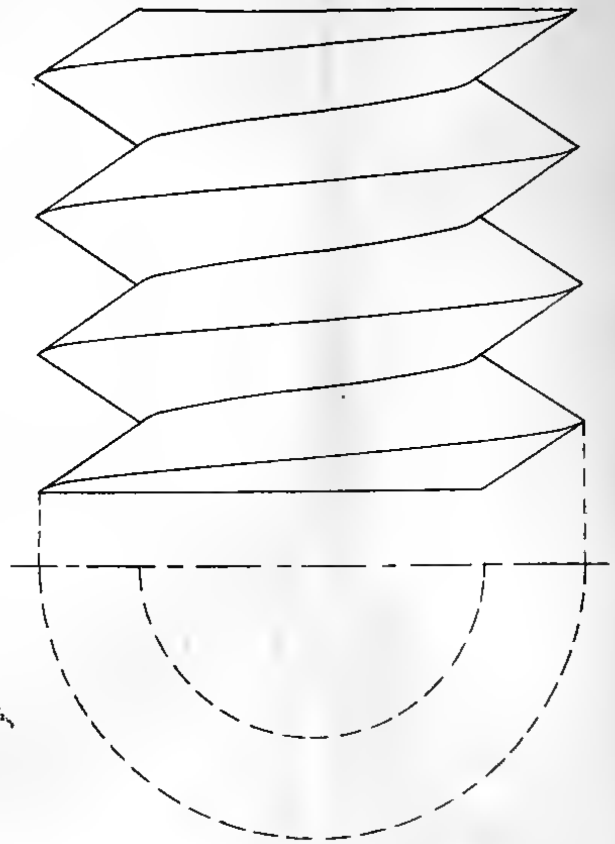


Fig.91

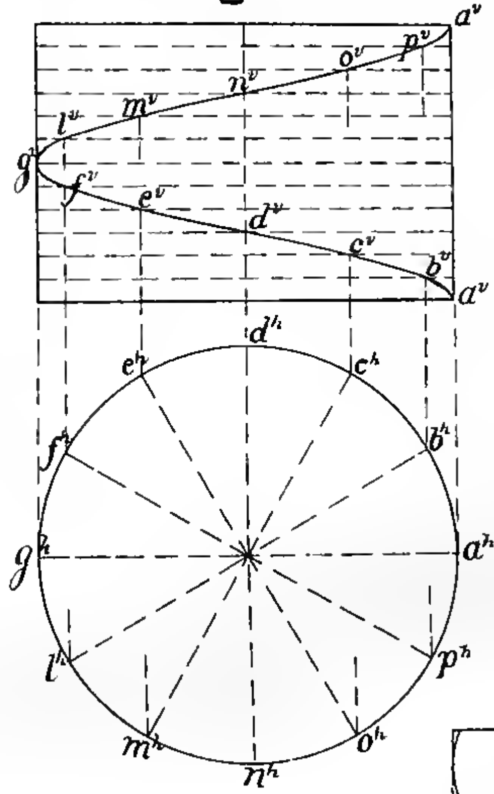
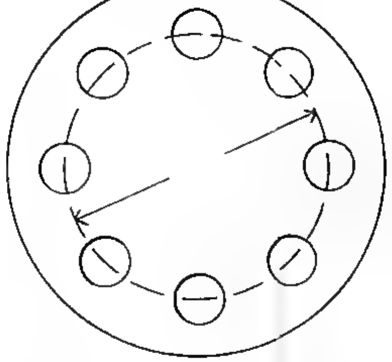


Fig.86



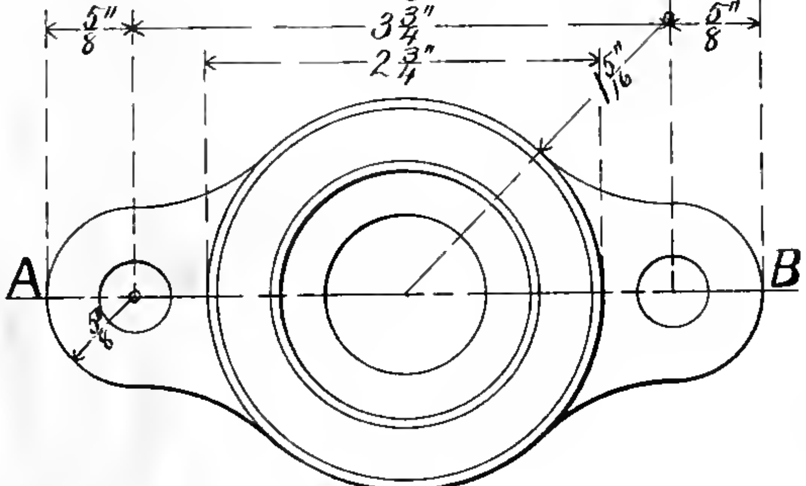


Fig.83

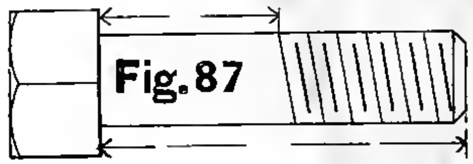


Fig.87

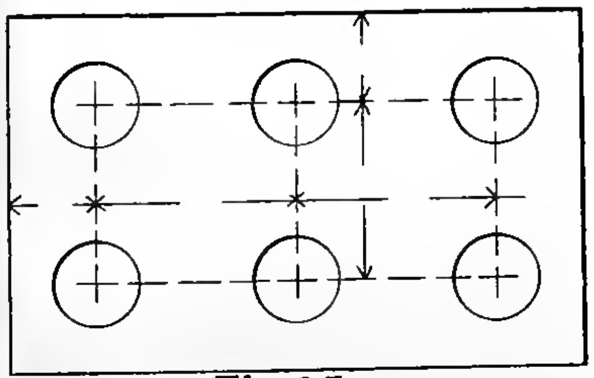


Fig.85

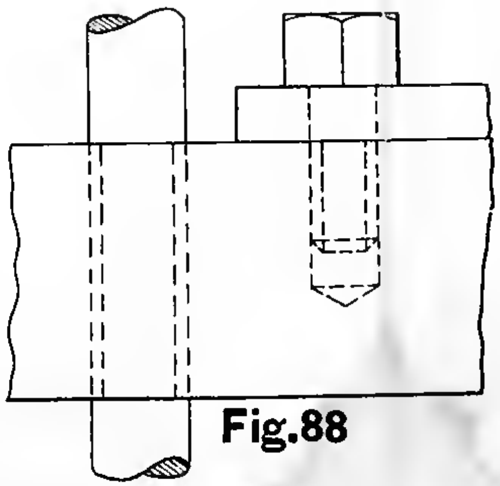


Fig.88

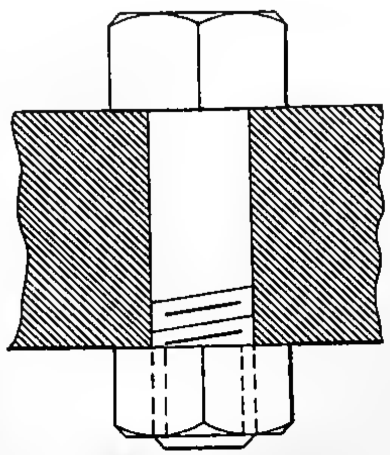


Fig.89

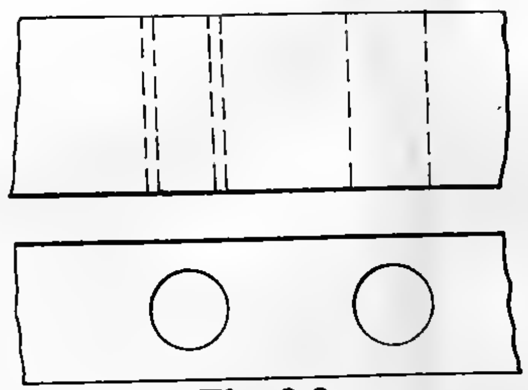


Fig.90

In making a sectional view on a line passing lengthwise through the centre of a shaft, bolt, or screw, it is generally unnecessary to represent the shaft, bolt, or screw in section, as the view is more clearly shown by leaving them in full. See Fig. 89.

In drawings where bolts or screws are shown by dotted lines do not dot in the threads, but represent them by double dotted lines, as shown in Fig. 89.

Represent a tapped hole as shown in Fig. 90.

The line on which a section is taken, as AB, Fig. 83, should be made the same as a centre line.

142. Although it is customary to represent a screw by straight lines, as shown in Fig. 87, it is sometimes desirable to make its actual projections, especially if the screw be a large one.

The thread of a screw is a curve which is called a helix. A cylindrical helix is generated by a point caused to travel round a cylinder, having, at the same time, a motion in the direction of the length of the cylinder,— this longitudinal motion bearing some regular prescribed proportion to the circular or angular motion. The distance between any two points which are nearest to each other, and in the same straight line parallel to the axis of the cylinder, is called the *pitch*,— in other words, the longitudinal distance traversed by the generating point during one revolution.

To draw the projections of a helix. Fig. 91.

The plan of the helix will be a circle. Divide this circle into any number of equal parts, in this case twelve; divide the pitch into the same number of equal parts. It is evident that when the point has moved one-twelfth the distance around the circumference, it has also moved in the direction of the axis one-twelfth of the pitch; when it has moved two-twelfths the

distance around it has moved two-twelfths of the pitch ; therefore, from the points of division, a^h, b^h, c^h, etc., in the plan draw vertical lines until they intersect horizontal lines drawn from the corresponding division of the pitch. And these intersections, a^v, b^v, c^v, etc., will be points on the vertical projection of the helix.

Fig. 92 shows a V-threaded screw in projection.

CHAPTER VIII.

EXAMPLES.

All polygons referred to in these Examples are regular polygons, unless otherwise stated.

1. Draw the two projections of a point $1\frac{1}{2}''$ from H and $1''$ from V.

2. Of a point lying in H and $\frac{3}{4}''$ in front of V.

3. Of a point lying in V and $1''$ above H.

4. Of a line $1''$ long, parallel to both V and H, $\frac{3}{4}''$ above H and $1''$ in front of V.

5. Of same line when it is perpendicular to V and $1\frac{1}{4}''$ above H, its back end being $\frac{1}{2}''$ in front of V.

6. Of same line when it is perpendicular to H and $1''$ in front of V, its lower end being $\frac{1}{4}''$ above H.

7. Of same line when it is parallel to H, $1''$ above H and making an angle of $30°$ with V, its back end being $\frac{1}{4}''$ in front of V.

8. Of same line when it is parallel to V, $\frac{3}{4}''$ in front of V and making an angle of $60°$ with H, its lower end being $\frac{1}{2}''$ above H.

9. Of same line lying in H and making an angle of $45°$ with V, its back end being $\frac{1}{2}''$ in front of V.

10. Of same line lying in V, parallel to and $1''$ above H.

11. Of same line when it is inclined at an angle of $60°$ with H, and whose horizontal projection makes an angle of $45°$ with GL, one end being $\frac{1}{4}''$ above H and $\frac{1}{2}''$ in front of V.

12. A wire $1\frac{1}{2}''$ long projects from a vertical wall at 60° with the surface, and is parallel to the ground and 1″ above it. Draw plan and elevation.

13. Find true length of a line given by its projections, as follows: One end is $\frac{1}{2}''$ from each plane and the other is 2″ above H, the horizontal projection of the line is $1\frac{1}{8}''$ long and makes an angle of 30° with GL.

14. Draw plan and elevation of a line $1\frac{1}{2}''$ long, lying in a profile plane, making an angle of 60° with H, whose lower end is $\frac{1}{4}''$ from V and $\frac{1}{2}''$ from H.

15. Of an oblique line, one end being above and in front of the other. Find its true length and angle it makes with H.

16. Of an oblique line, one end being behind and above the other. Find its true length and angle it makes with V.

17. Of a line which slopes downward, backward, and to the right. Find its true length by revolving parallel to V.

18. Of a line which slopes downward, forward, and to the left. Find its true length by revolving parallel to H.

19. Of two lines which are parallel in space and slope downward, forward, and to the right.

20. Of a line 2″ long, sloping downward, forward, and to the right, one end being $1\frac{1}{2}''$ above H and $\frac{1}{2}''$ in front of V, the other end $\frac{1}{2}''$ above H and $1\frac{1}{4}''$ in front of V.

21. Draw plan and elevation of a rectangular card $\frac{3}{4}''$ x $1\frac{1}{4}''$ which is perpendicular to H, parallel to V, and $\frac{3}{8}''$ in front of V; its short sides are parallel to H and the lower one is $\frac{1}{4}''$ above H. Revolve this card forward about its left-hand edge (like a door on its hinges) through angles of 30°, 45°, 60°, and 90°, and construct corresponding plans and elevations.

22. Of same card when it is lying on H with its long sides parallel to V and $\frac{1}{4}''$ in front of V. Revolve card about right-hand horizontal edge (like a trap-door on its hinges) through

angles of 30°, 45°, 60°, and 90°, and construct corresponding projections.

23. Of same card when, besides making the angles of 0°, 30°, 45°, 60°, and 90° with H, as in last example, the horizontal projection of its long edges, in all its different positions, makes an angle of 30° with GL backward and to the right.

24. Of same card when it is parallel to V and $\frac{1}{4}''$ in front of V, one of its diagonals being parallel to H. Revolve this card through an angle of 60° about a vertical axis, and construct its corresponding projections.

25. Of same card when, besides making an angle of 60° with V, as in last example, the vertical projection of the diagonal which was parallel to H makes an angle of 45° with GL.

26. Of a card $\frac{7}{8}''$ square resting on H, with one diagonal parallel to V and 1″ in front of V, then raise left-hand end of diagonal until it makes an angle of 45° with H and construct corresponding projections.

27. Of the same card when, besides making an angle of 45° with H, as in last example, the horizontal projection of the diagonal makes an angle of 30° with GL.

28. Same as example 27, except that the horizontal projection of the diagonal makes an angle of 90° with GL.

29. Of an hexagonal card whose sides are $\frac{3}{4}''$ long, when one of its long diameters is parallel to V and 1″ in front of V, one end resting on H. The surface of the card is perpendicular to V and makes an angle of 30° with H.

30. Of same card when its diameter, besides making an angle of 30° with H, as in last example, has its horizontal projection inclined at an angle of 60° with GL.

31. Of a pentagonal card whose surface is perpendicular to H and makes an angle of 45° with V, one edge being perpendicular to H and resting against V. The diameter of the circumscribed circle is $1\frac{1}{2}''$.

32. Of same card when, besides making an angle of 45° with V, as in last example, it has been revolved through an angle of 30°.

33. Of an octagonal card resting on one of its edges, with its surface perpendicular to V and inclined at an angle of 60° with H. The diameter of the inscribed circle is $1\frac{1}{2}''$.

34. Of same card when it is inclined at same angle with H as in last example, and the horizontal projection of the edges which were perpendicular to V makes an angle of 45° with GL.

35. Of an isosceles triangle situated in a profile plane, the base making an angle of 15° with H, its back corner being $\frac{3}{8}''$ above H and $\frac{7}{8}''$ in front of V, and lower than its front corner. The base of triangle is $1\frac{1}{8}''$ and altitude $1''$.

36. Of an hexagonal card whose surface is perpendicular to both V and H. Its long diameter is $1''$. Two of its edges are perpendicular to H. Centre of card is $1''$ above H, and $1\frac{1}{4}''$ in front of V.

37. Of a circular card $3''$ in diameter, perpendicular to H and making an angle of 30° with V, resting on H, back edge $1''$ from V.

38. Of same card after it has been revolved through an angle of 45°, keeping the same angle with V as in last example.

39. Of a circular card $3''$ in diameter, perpendicular to H and making an angle of 45° with V. Solve as explained in Art. 61.

40. Draw plan and elevation of a cube of $1''$ edge resting on H, $\frac{1}{4}''$ in front of V, with two faces parallel to V. Find the true size of the angle which the diagonal, which slopes downward, backward, and to the right, makes with V and also with H.

41.* Of a rectangular prism whose base is $\frac{3}{4}''$ x $1''$ and length is $1\frac{1}{2}''$, resting with its base against V, its lower left-hand face making an angle of 60° with V.

Shade lines are to be put in in all the examples where there are any.

42. Of a cylinder resting with its base on H; diameter of cylinder is $1''$ and its length is $1\frac{1}{2}''$. The axis is $\frac{3}{4}''$ in front of V.

43. Of a cone resting with its base parallel to and $\frac{1}{4}''$ in front of V. The diameter of base is $1\frac{1}{4}''$ and its height is $1\frac{3}{4}''$.

44. Of a heptagonal prism resting with its base on H, one of its faces making an angle of 15° with V. The diameter of the circumscribed circle about base is $1''$ and the height of prism is $1\frac{3}{4}''$.

45. Of an octagonal pyramid resting with its base on H, with two of the edges of the base making an angle of 30° with GL. The diameter of the circle inscribed in the base is $1\frac{1}{2}''$ and the altitude is $1''$.

46. Draw plan and two elevations of a square prism with its axis parallel to both V and H; the axis is $1''$ above H and $1\frac{1}{4}''$ in front of V, base of prism is $\frac{3}{4}''$ square, and the length is $1\frac{1}{2}''$; the upper left-hand long face makes an angle of 30° with H.

47. Draw plan and elevation of same prism when its axis is parallel to H and makes an angle of 45° with V, backward and to the left.

48. Draw plan and two elevations of an hexagonal prism $2''$ long, diameter of circumscribed circle about base is $1\frac{1}{4}''$, the axis is parallel to H and makes an angle of 30° with V, backward and to the right; lower edge of prism rests on H, and the lower right-hand face makes an angle of 20° with H.

* The base is supposed to be at right angles to the axis in all the prisms, pyramids, cylinders, and cones, unless otherwise stated.

49. Draw plan and two elevations of a circular cylinder, $3''$ in diameter and $2\frac{1}{2}''$ long, with a circular hole through it $1\frac{1}{2}''$ in diameter; axis is parallel to both V and H.

50. Draw plan and elevation of same cylinder resting with its base on H.

51. Of same cylinder when lying on H, with its axis parallel to H and making an angle of $60°$ with V.

52. Draw plan and two elevations of a pile of blocks located as follows: the lowest one is $3''$ long, $1\frac{1}{4}''$ wide, and $\frac{1}{2}''$ thick, it rests with its wide face on H, its long edge making an angle of $30°$ with V, backward and to the right; on top of this a second block rests, equal in width and thickness to the first but $1''$ shorter; on this a third block rests, of the same width and thickness as the others, but $1''$ shorter than the second; these blocks are placed symmetrically.

53. Draw plan and elevation of a pyramid formed of four equilateral triangles of $2''$ sides, when one edge of the base is at $30°$ with V.

54. Of an hexagonal prism standing with its base on H, two of its faces making angles of $20°$ with V; diameter of circumscribed circle about base is $8''$, length of prism is $10''$, scale $3''=1'$, or $\frac{1}{4}$ size.

55. Of same prism when axis is parallel to V and makes an angle of $60°$ with H and slopes downward to the left. Scale $3''=1'$.

56. Of same prism when its axis, besides making an angle of $60°$ with H, has its horizontal projection inclined at an angle of $60°$ with GL; axis of prism slopes downward, forward, and to the left. Scale $2''=1'$.

57. Of a pentagonal pyramid resting with its base on H, one edge of base perpendicular to V, diameter of circumscribed circle $16''$, height of pyramid $20''$. Find its shadow. Scale $1\frac{1}{2}''=1'$, or $\frac{1}{8}$ size.

58. Of same pyramid when its axis is parallel to V and slopes downward to the left, making an angle of 75° with H. Find its shadow. Scale $1\frac{1}{2}'' = 1'$.

59. Of same pyramid when its axis, besides making an angle of 75° with H, has its horizontal projection inclined at an angle of 30° with GL, so that axis slopes downward, backward, and to the right. Find its shadow. Scale $1\frac{1}{2}'' = 1'$.

60. Of a cone resting with its base on H, diameter of base 20″, height of cone 2′. Find its shadow. Scale $1\frac{1}{2}'' = 1'$.

61. Of same cone when resting with an element on H, with its axis parallel to V. Find its shadow. Scale $1\frac{1}{2}'' = 1'$.

62. Of same cone with an element on H, and axis making an angle of 45° with GL. Find its shadow. Scale $1\frac{1}{2}'' = 1'$.

63. Of same cone with an element on H, and axis lying in a profile plane, sloping downward and backward. Find its shadow. Scale $1\frac{1}{2}'' = 1'$.

64. Of the frustum of an octagonal pyramid resting with its base on H, long diameter of lower base is 3″ and of the upper base is 2″, the height of frustum is 3″, the front left-hand edge of base makes an angle of 15° with GL, backward to the left. There is a hole 1″ square through the centre of frustum, whose axis is coincident with axis of frustum; two sides of the hole make angles of $7\frac{1}{2}°$ with GL.

65. Of same frustum when its axis is parallel to V and makes an angle of 60° with H and slopes downward to the left.

66. Of same frustum when its axis, besides making an angle of 60° with H, has its horizontal projection inclined at an angle of 30° with GL, and slopes downward, forward, and to the left.

67. Of the skeleton frame of a box 3′ long, 2′ wide, and 2′ high, the joists being all $2\frac{1}{2}''$ square. The frame rests on H with its long sides parallel to V. Do not show joints in framing. Scale $1'' = 1'$.

68. Of same frame still resting on H with its long sides making an angle of 50° with V, backward to the left. Scale $1''=1'$.

69. Revolve the elevation obtained in Example 68 through an angle of 30° (in either direction), and construct corresponding plan. Scale $1''=1'$.

70. Of a double cross standing on its base, one arm parallel to both V and H; upright piece is $1'-8''$ square and $10'-8''$ high, each arm is $1'-8''$ square and $7'$ long (out to out), their top surfaces are $2'-8''$ below the top of upright. Scale $\frac{3}{8}''=1'$.

71. Of same cross when its axis is parallel to V and makes an angle of 60° with H. Scale $\frac{3}{8}''=1'$.

72. Of same cross when its axis, besides making an angle of 60° with H, has its horizontal projection inclined at an angle of 60° with H, sloping downward, forward, and to the left. Scale $\frac{3}{8}''=1'$.

73. Of a pyramid resting on its apex with axis perpendicular to H and $2'-6''$ in front of V, its base is an equilateral triangle of $2'-8''$ sides and its altitude is $11''$. The left-hand edge of base is perpendicular to V. Find its shadow. Scale $\frac{3}{4}''=1'$.

74. Of same pyramid when its axis is parallel to V and makes an angle of 60° with H, and slopes downward to the right. Find its shadow. Scale $\frac{3}{4}''=1'$.

75. Of same pyramid when its axis, besides making an angle of 60° with H, has its horizontal projection inclined at an angle of 30° with GL, so that it slopes downward, forward, and to the left, its apex being $1'-6''$ from V, still resting on H. Find its shadow. Scale $\frac{3}{4}''=1'$.

76. There is a solid formed of two equal square pyramids of $2''$ base and $3''$ altitude, which are united by their bases. Draw plan and elevation when the object rests with one of its triangular faces on H, its axis being parallel to V and $2\frac{1}{2}''$ in front of V. Find its shadow.

77. Of same object still resting on one of its faces, when the horizontal projection of the axis makes an angle of 45° with GL, and slopes downward, backward, and to the right. Find its shadow.

78. Draw projections of a pentagonal prism whose length is $2\frac{1}{2}''$ and radius of circumscribed circle about the end is $\frac{7}{8}''$; the prism rests with one of its long edges on H, which makes an angle of 60° with V, backward to the left, and whose front end is $3''$ from V. The lower left-hand long face makes an angle of 15° with H.

Also, of a triangular pyramid resting on its base on H, with its axis $1\frac{3}{4}''$ to the left of the point located in prism, and $4''$ in front of V, diameter of circumscribed circle is $2''$, the altitude of pyramid is $3\frac{1}{2}''$, right-hand edge of base is perpendicular to V. Find shadow of prism on H and V, also of pyramid on H and on prism.

79. Find the shadow on H of a card $\frac{5}{8}''$ square, parallel to H, and $\frac{7}{8}''$ above H, two edges making angles of 30° with V.

80. Of same card on H, when it is parallel to V, $2''$ in front of V, two edges parallel to H, lowest edge $\frac{1}{4}''$ above H.

81. Of same card on V and H, lying in a profile plane, two edges perpendicular to H, back edge $\frac{1}{2}''$ in front of V and lowest edge $\frac{1}{4}''$ above H.

82. Of an hexagonal card, parallel to V, $\frac{1}{2}''$ in front of V, two edges parallel to H, centre of hexagon $1\frac{1}{4}''$ above H, long diameter $1''$.

83. Of same card parallel to H, $1\frac{1}{8}''$ above H, centre $1''$ in front of V, two edges parallel to V.

84. Of a circular card, $1\frac{1}{4}''$ in diameter, parallel to V, $1\frac{1}{8}''$ in front of V, centre $1''$ above H.

85. Of an hexagonal card whose surface is perpendicular to V and H, two of its edges perpendicular to H, centre of

hexagon $\frac{15}{16}''$ above H, $1\frac{1}{16}''$ in front of V, diameter of inscribed circle $1''$. In constructing projections of hexagon revolve it about a vertical axis through centre.

86. Of a cube of $\frac{5}{8}''$ sides, parallel to V and H, $1\frac{1}{8}''$ above H, and $\frac{1}{4}''$ in front of V.

87. Of a square prism standing on H, each face $\frac{5}{8}'' \times 1\frac{1}{2}''$, two faces parallel to V, $\frac{1}{8}''$ in front of V.

88. Of same prism still standing on H, turned so that two faces make angles of 30° with V, backward to the right.

89. Of a cylinder $\frac{3}{4}''$ in diameter and $1\frac{3}{4}''$ high, with base resting against V $1''$ above H.

90. Of a cone $1\frac{1}{4}''$ high, base $\frac{3}{4}''$ in diameter, standing on H, with axis $1\frac{3}{4}''$ in front of V.

91. Of a line located as shown in Example 20.

92. Of a card located as shown in Example 36.